
EXPRESSIONS OF CHIRAL DYNAMICS IN HADRONS AND NUCLEI

Daniel Phillips
Ohio University



Research supported by the US Department of Energy

CHIRAL DYNAMICS

M (MeV)



M_N 939

ω ————— 770
 ρ

χ PT

π ————— 138

CHIRAL DYNAMICS

- For probe energies~a few hundred MeV, simplifications of the rich QCD dynamics emerge: processes dominated by π s and Δ s

- BUT, interplay with other QCD states crucial in detailed comparisons with data

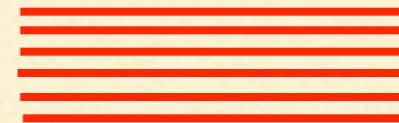
- EFTs provide a way to organize the theory: expansion in $M_{lo}/M_{hi} \rightarrow$ theory uncertainties

- Delineate this interplay precisely

- Possible to obtain high-accuracy hadronic numbers. Key to Intensity Frontier searches for BSM physics and determination of SM parameters.

- Here emphasis on EM probes

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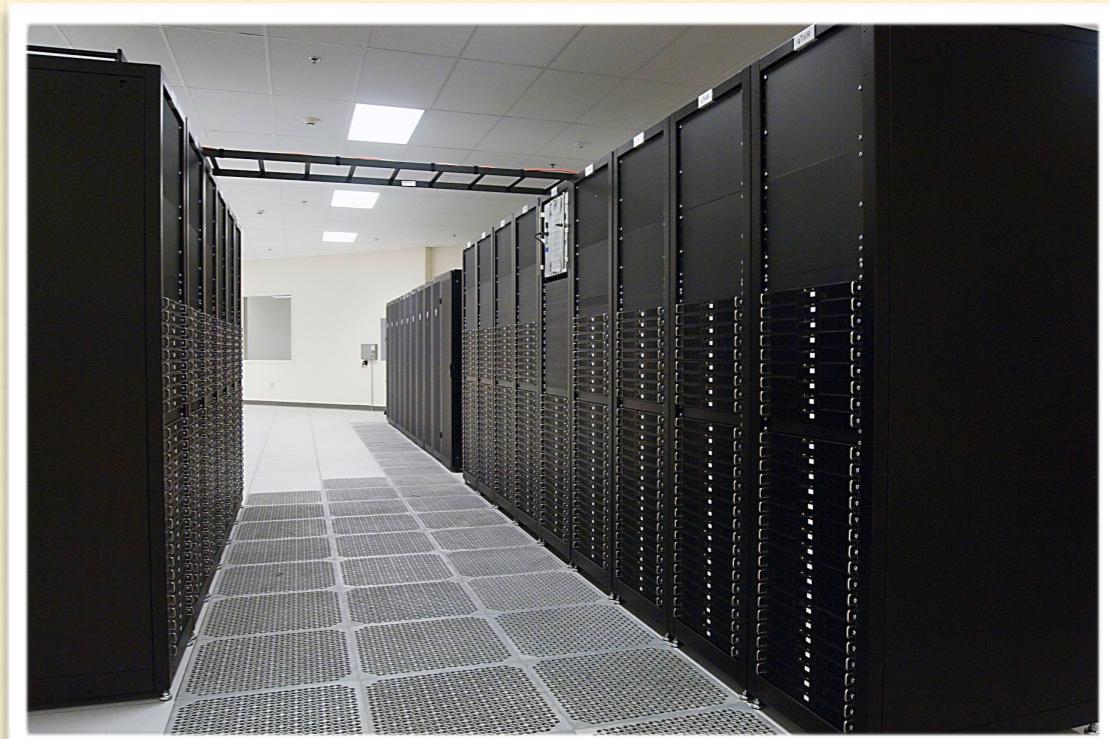
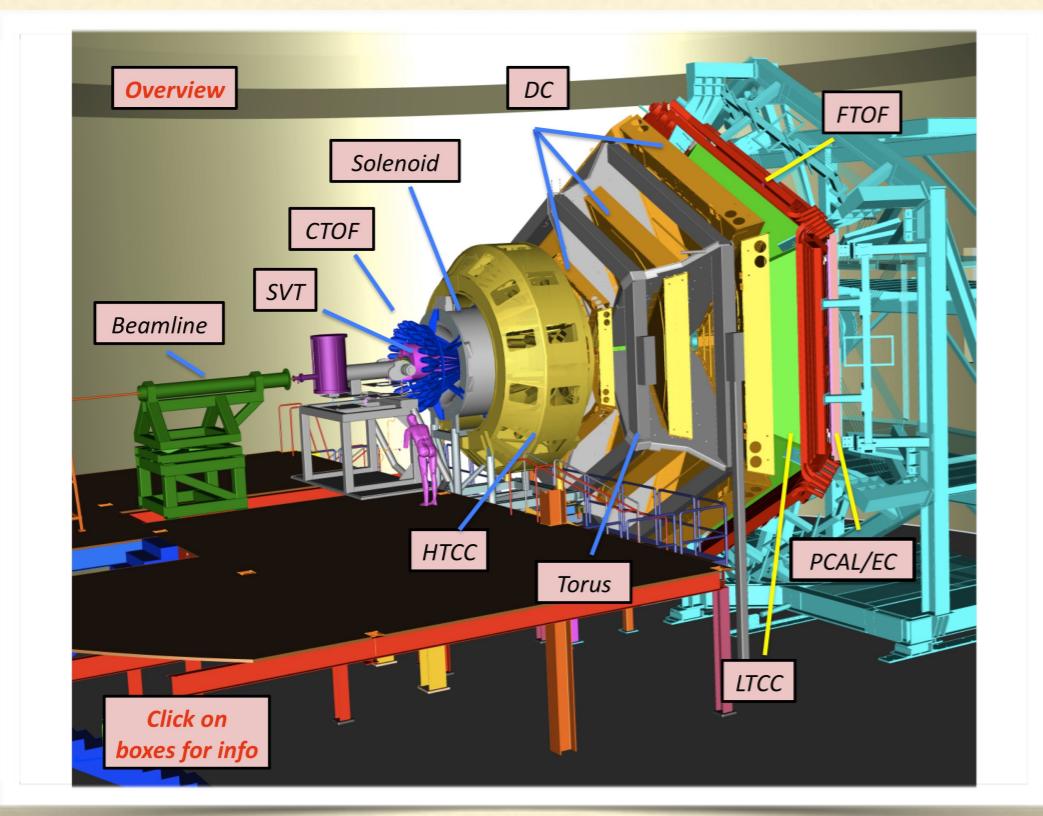
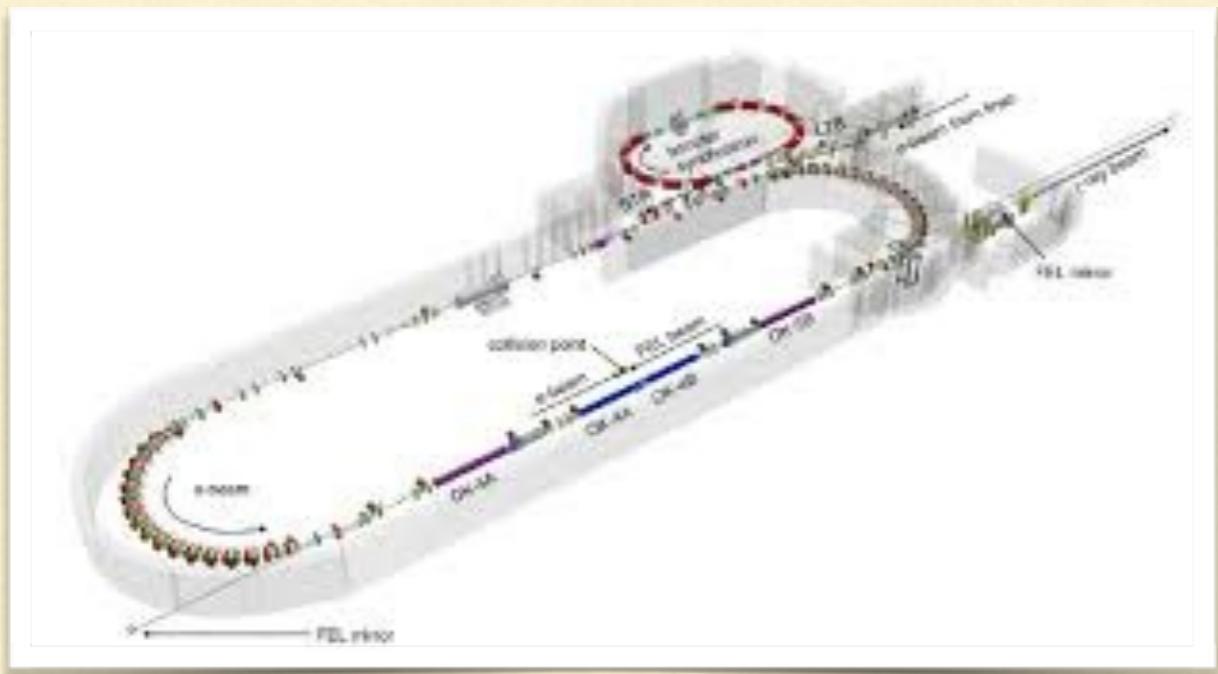
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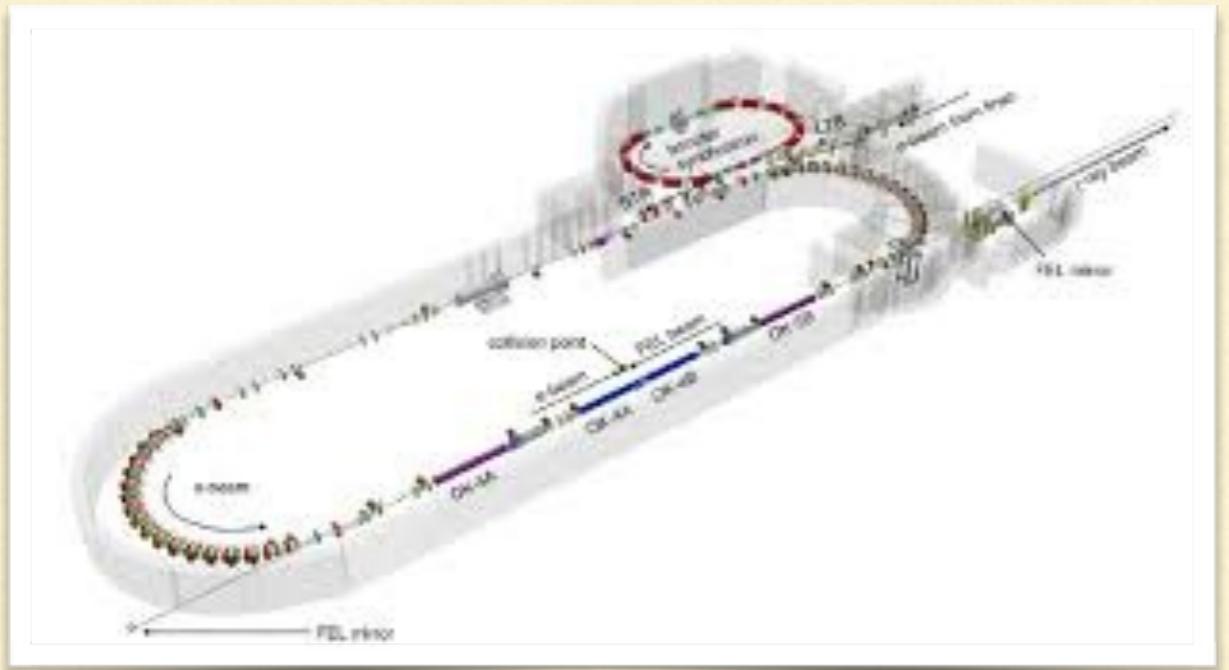
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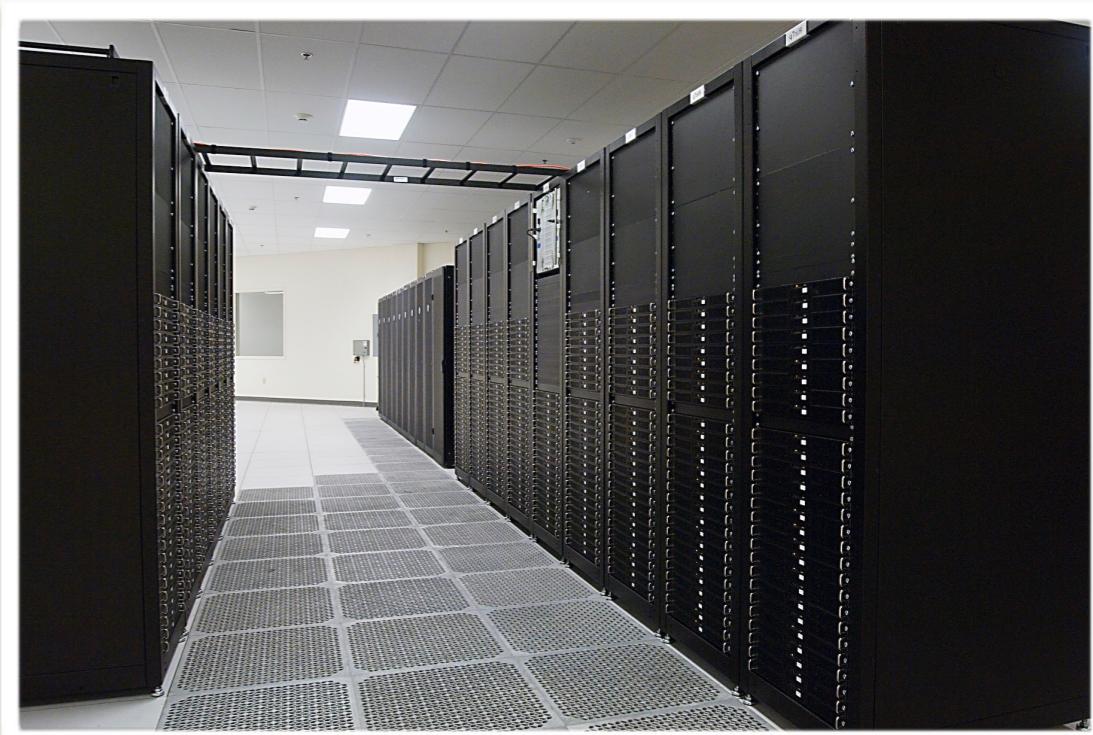
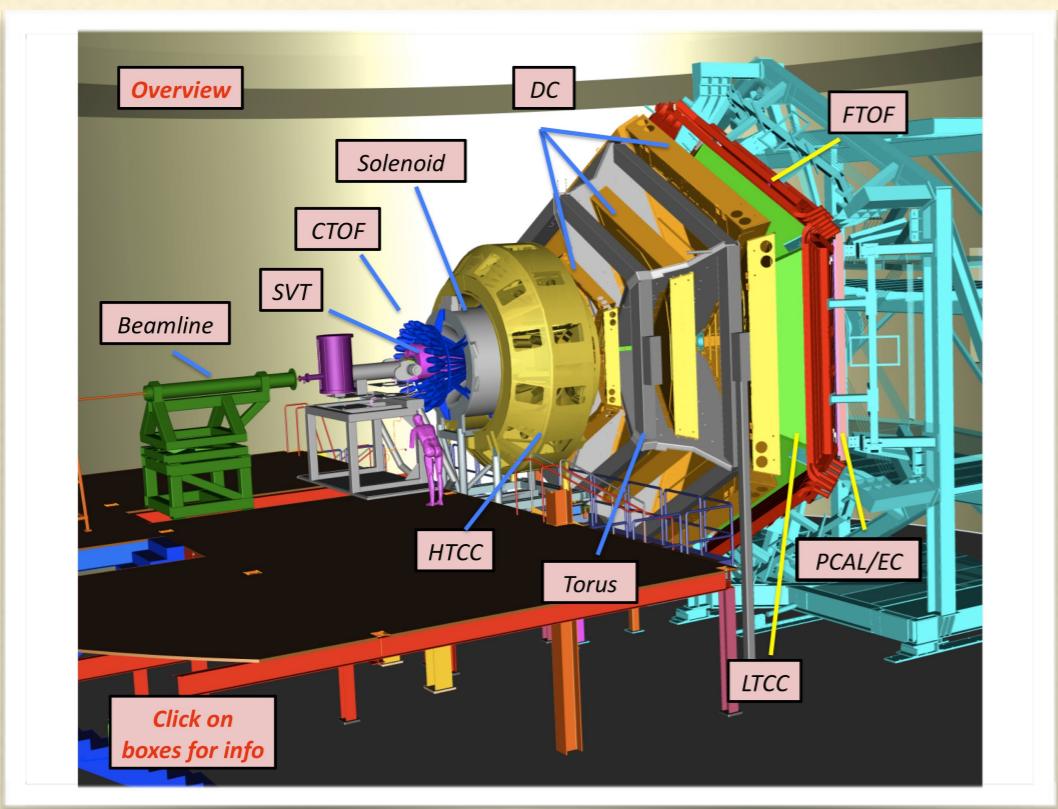
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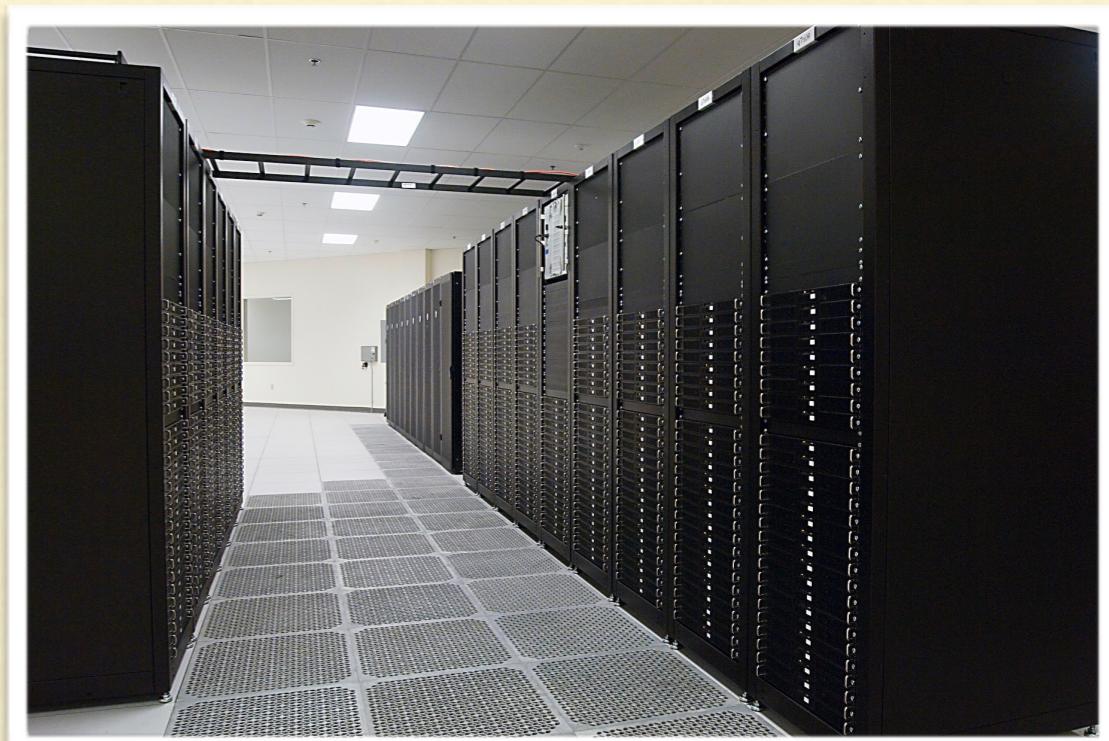
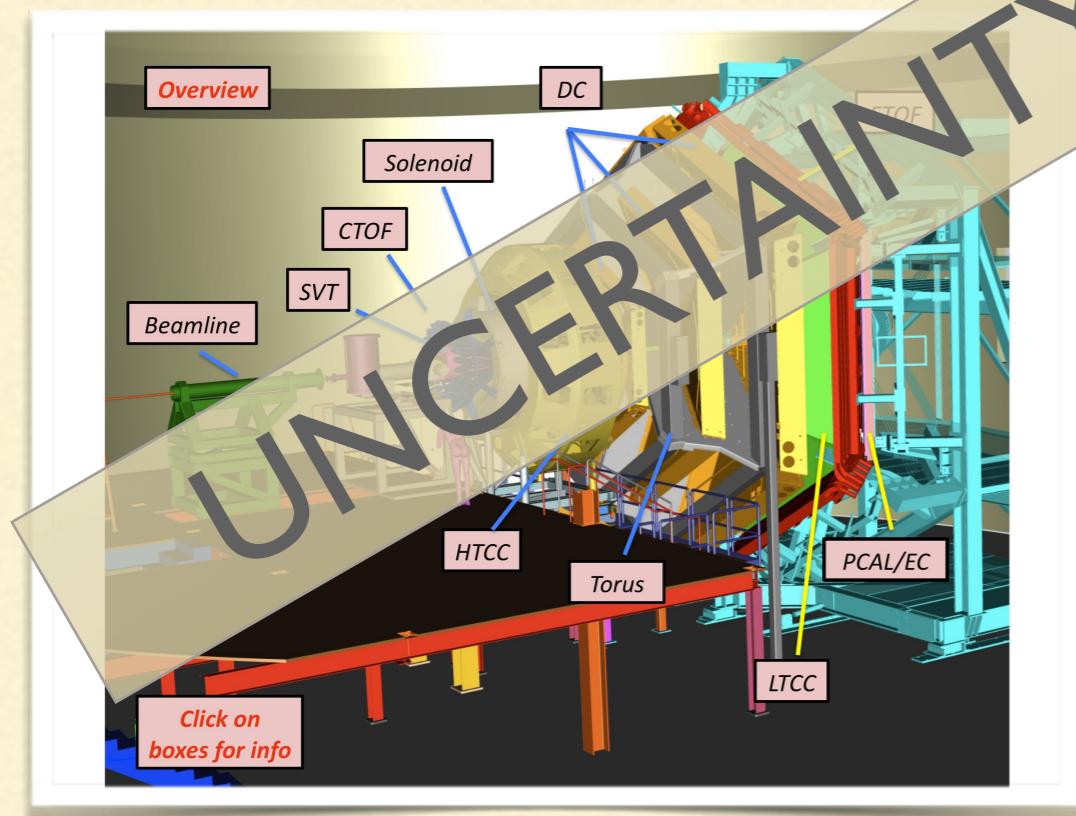
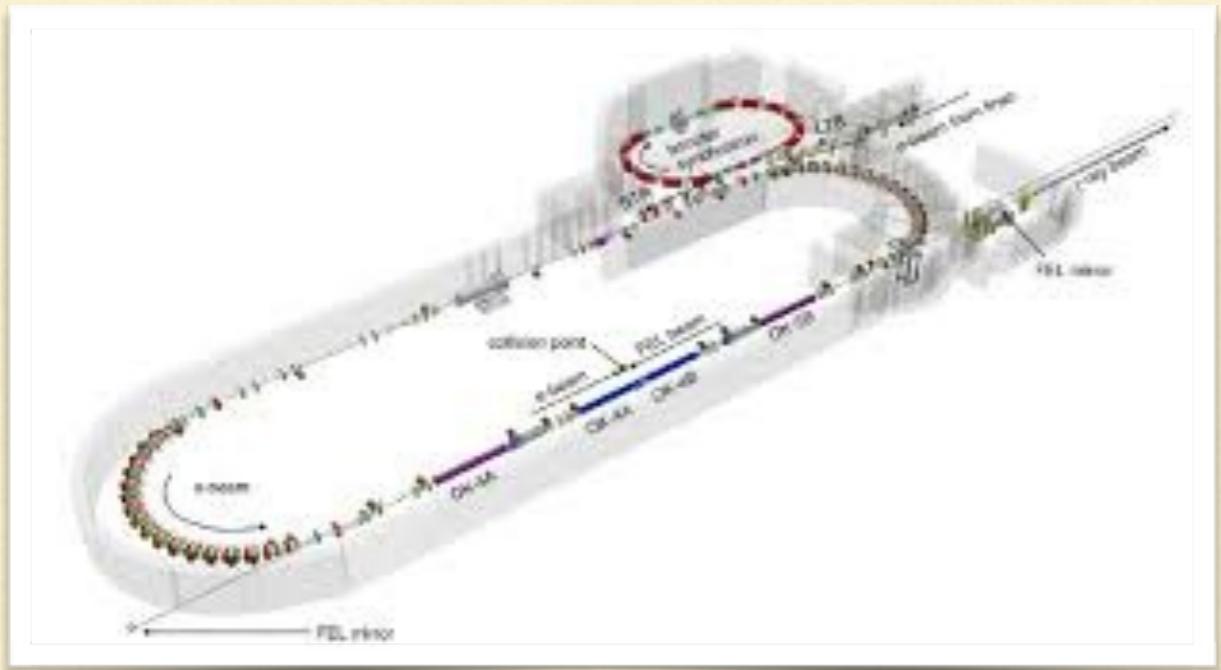






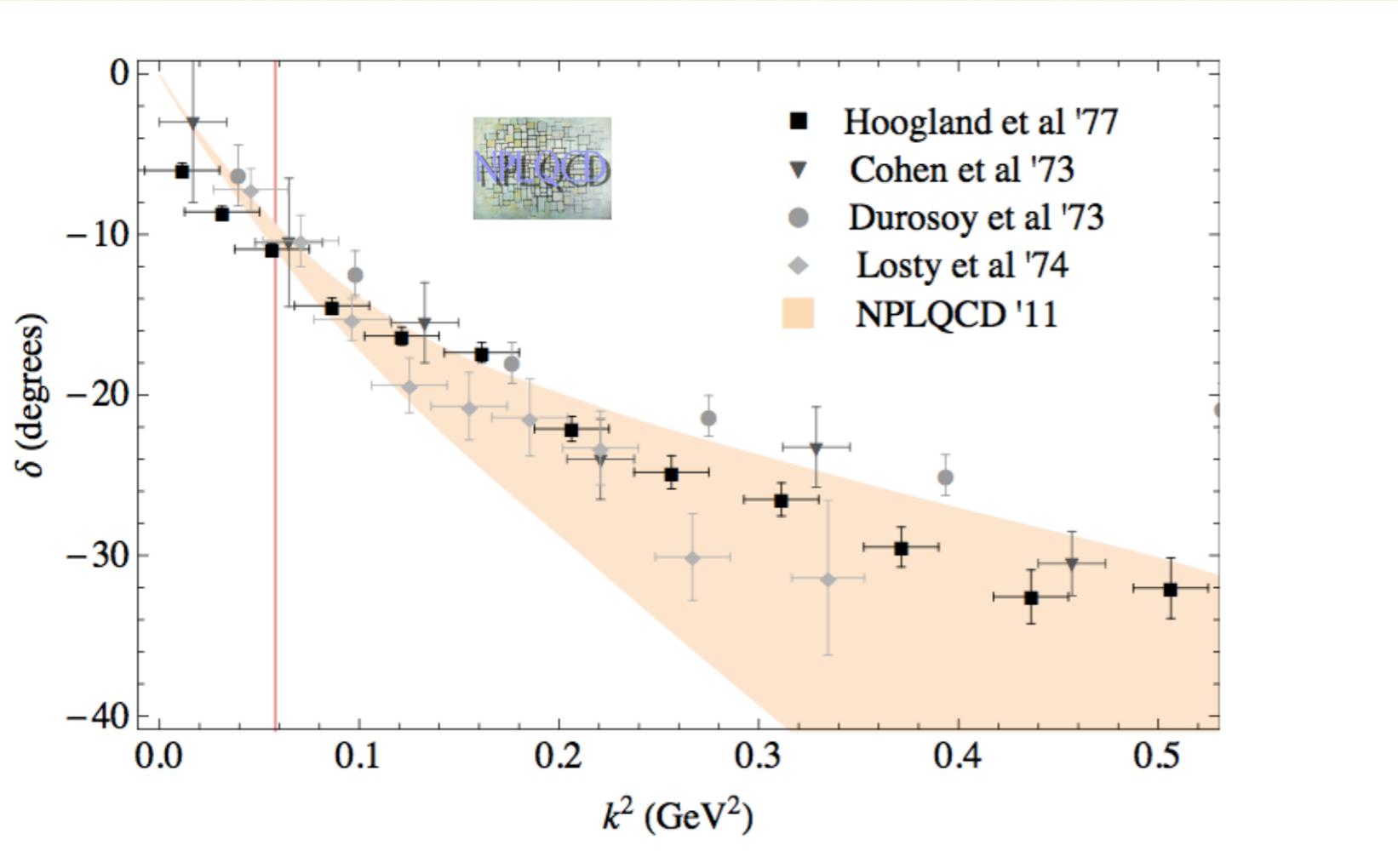
EFT





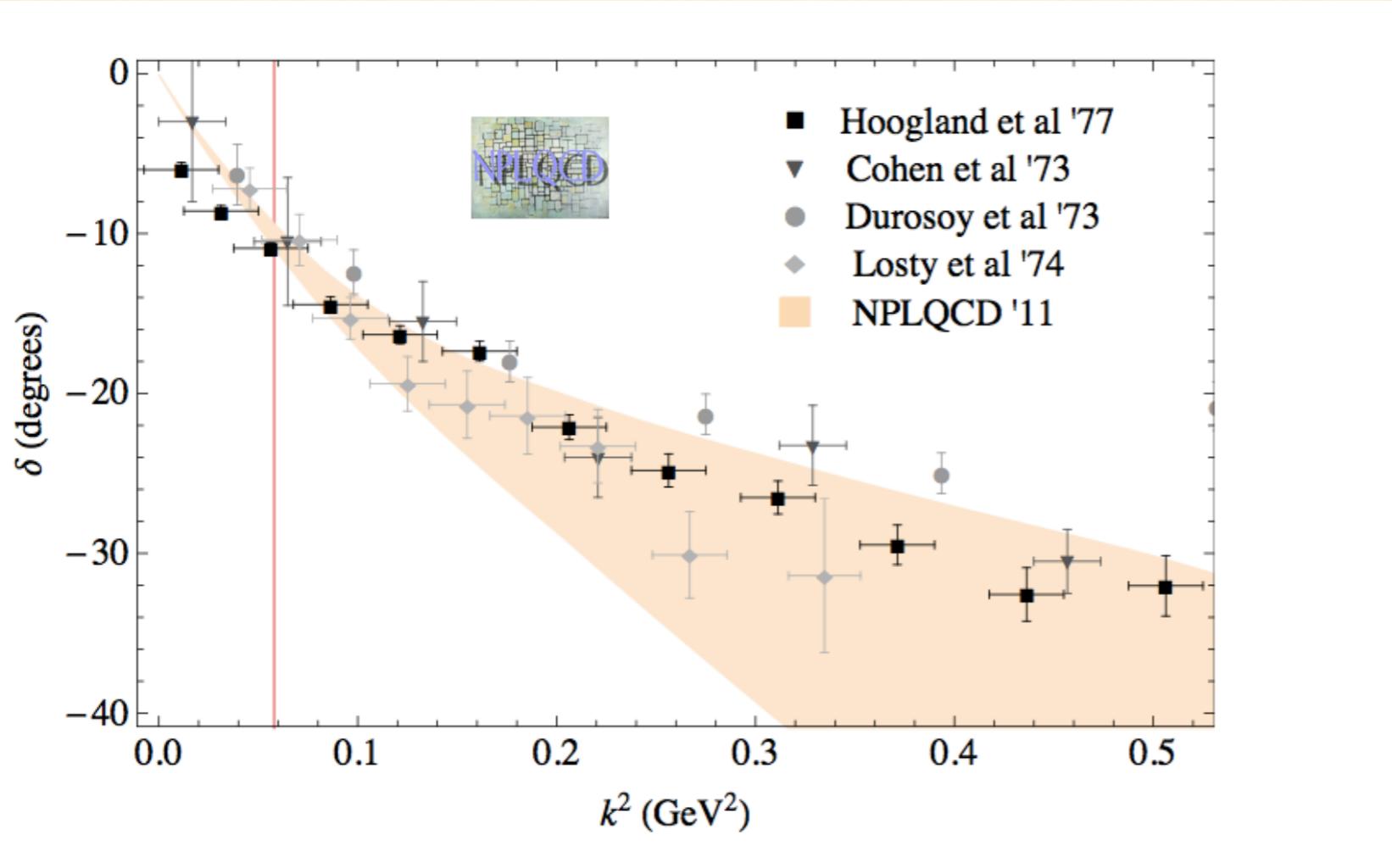
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HSC, NPLQCD (2011)



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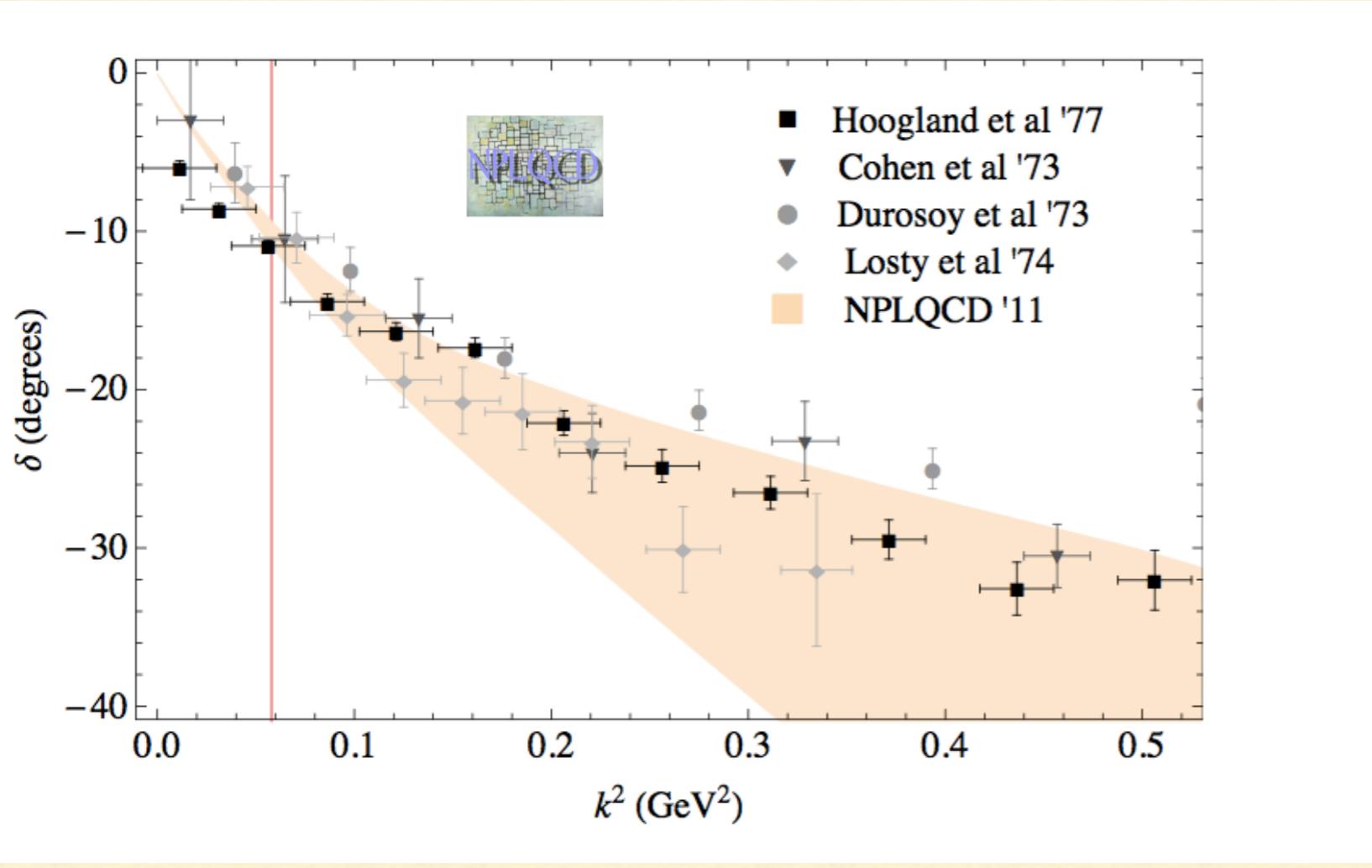
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- Anisotropic $n_f=2+1$ clover. $m_\pi \approx 390$ MeV
- Several energy levels in the box measured, at different L. Lüscher for δ

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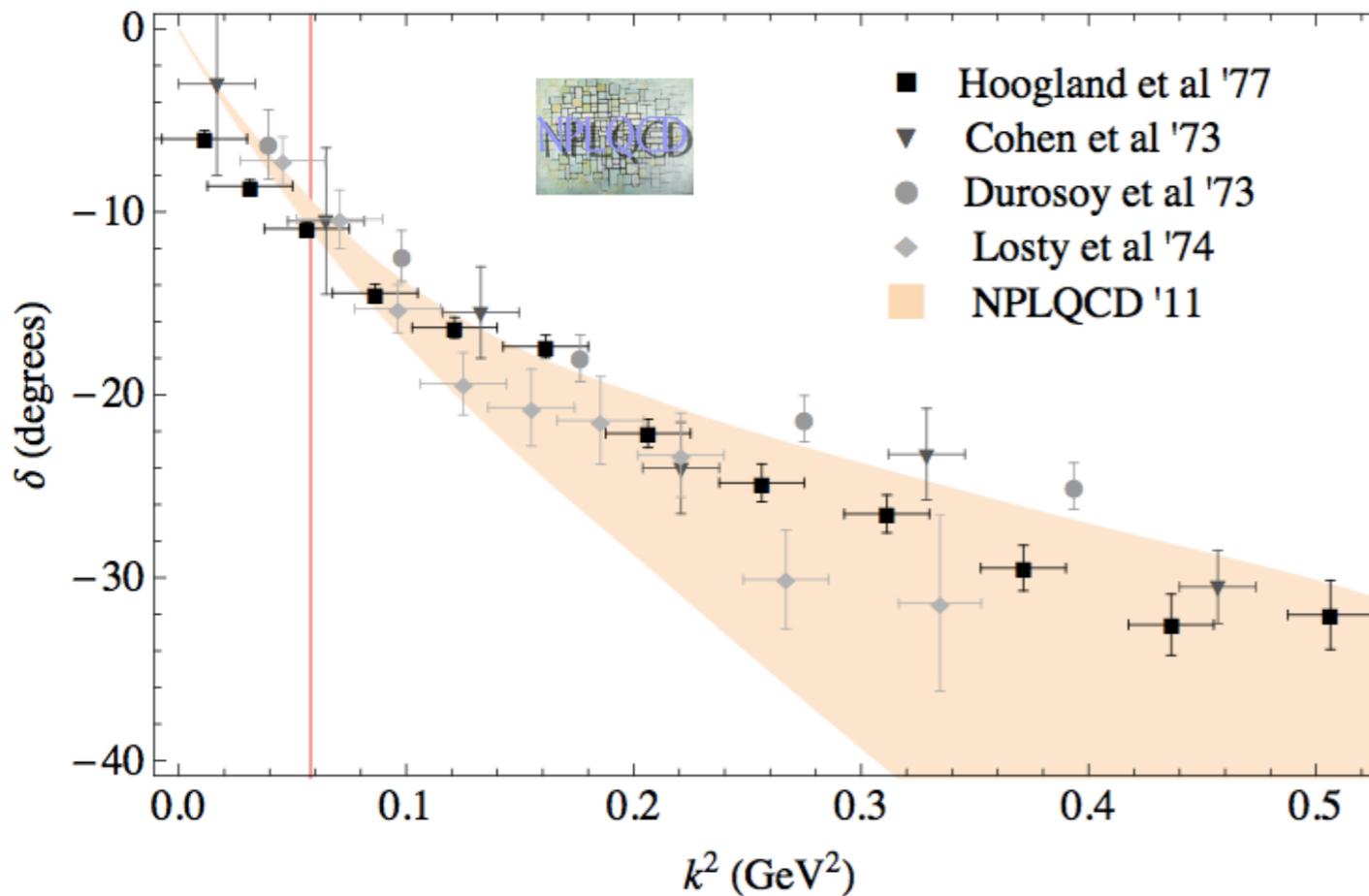
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NA48:
 $m_\pi a = 0.0429(44)(28)$

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Roy equations:

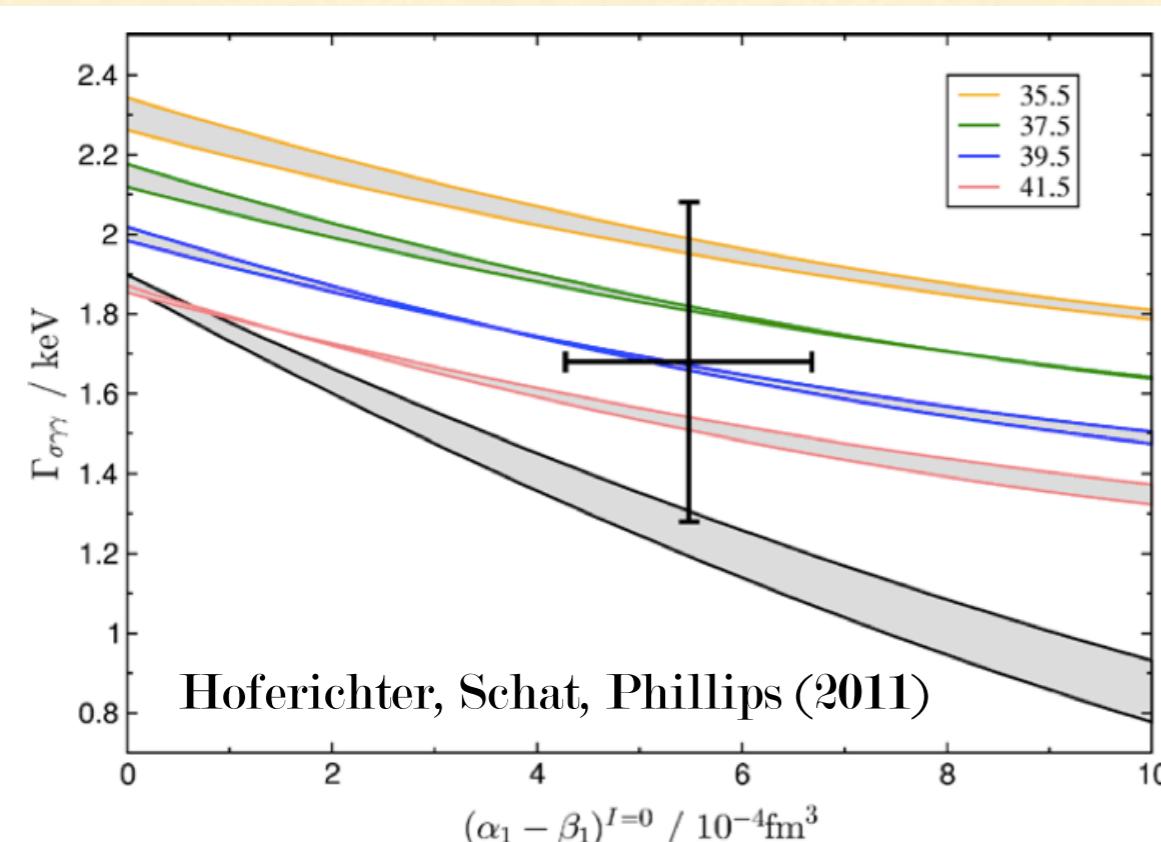
$$m_\pi a = 0.0444(1)$$

Colangelo, Gasser, Leutwyler (2001)

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$\gamma\gamma \rightarrow \pi\pi$ AND π POLARIZABILITIES

- Roy equations=dispersion relations + partial-wave expansion + crossing symmetry + unitarity \Rightarrow Coupled set of equations for partial-wave amplitudes
- Ensuring low-energy amplitude matches to χ PT and using data at energies > 1 GeV as input gives accurate $\pi\pi$ phase shifts over a wide energy range
- Sigma resonance pole position: $M_\sigma = 441^{+16}_{-8}$ MeV; $\Gamma_\sigma = 554^{+18}_{-25}$ MeV
Caprini, Colangelo, Leutwyler (2006)
- Roy-Steiner equations for $\gamma\gamma \rightarrow \pi\pi$. Subtractions suppress left-hand cut



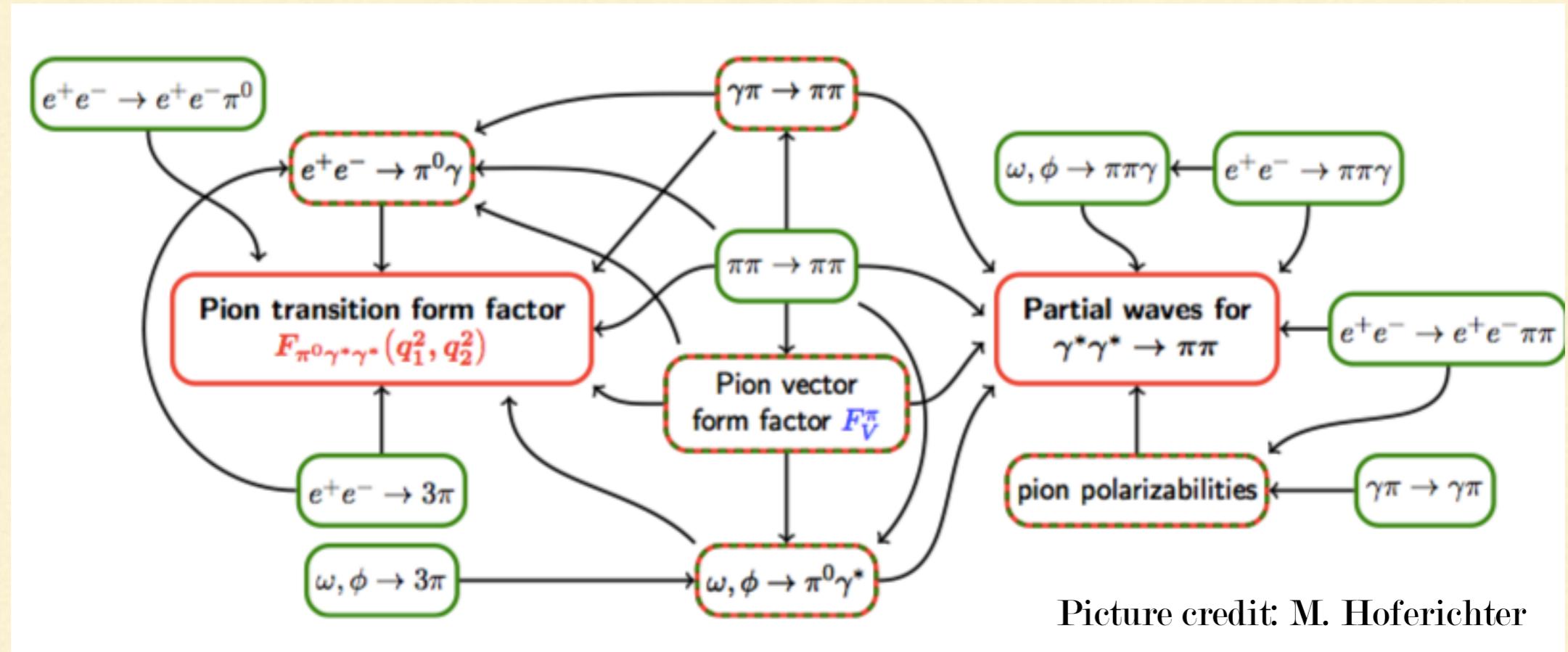
χ PT input for π polarizabilities \Rightarrow
 $\Gamma_{\sigma\gamma\gamma} = (1.7 \pm 0.4) \text{ keV}$

COMPASS:
 $\alpha_\pi = (1.9 \pm 0.7_{\text{stat.}} \pm 0.8_{\text{sys.}}) \times 10^{-4} \text{ fm}^3$

JLab Hall D: PR-13-008, A- rating
Spokesperson: R. Miskimen (U. Mass.)

DOING BETTER ON HADRONIC LBL

Data + dispersion relations with chiral constraints

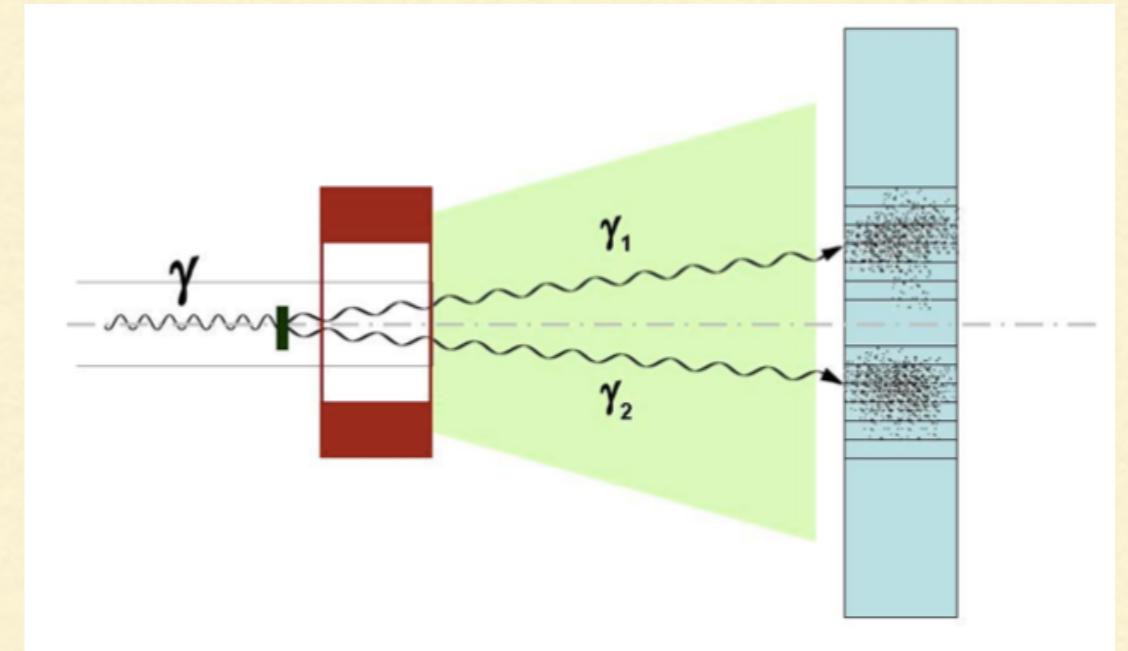
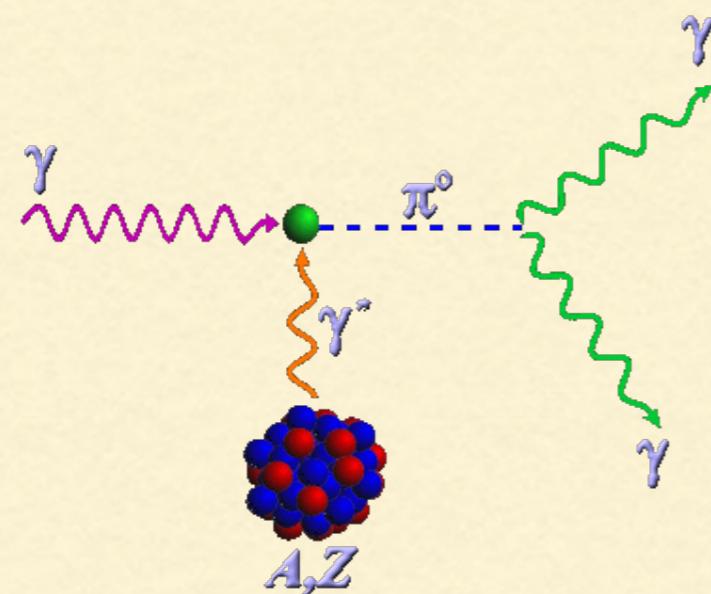


UNCERTAINTY QUANTIFICATION

- Reconstruction of $\gamma^*\gamma^* \rightarrow \pi\pi, \pi^0$: combine experiment and theory constraints
- Next step: $\eta, \eta', K\bar{K}$, multi-pion channels, pQCD constraints for high energy
- Measured resonance parameters play a key role in constraining this analysis

PRIMEX AND PRIMEX-II

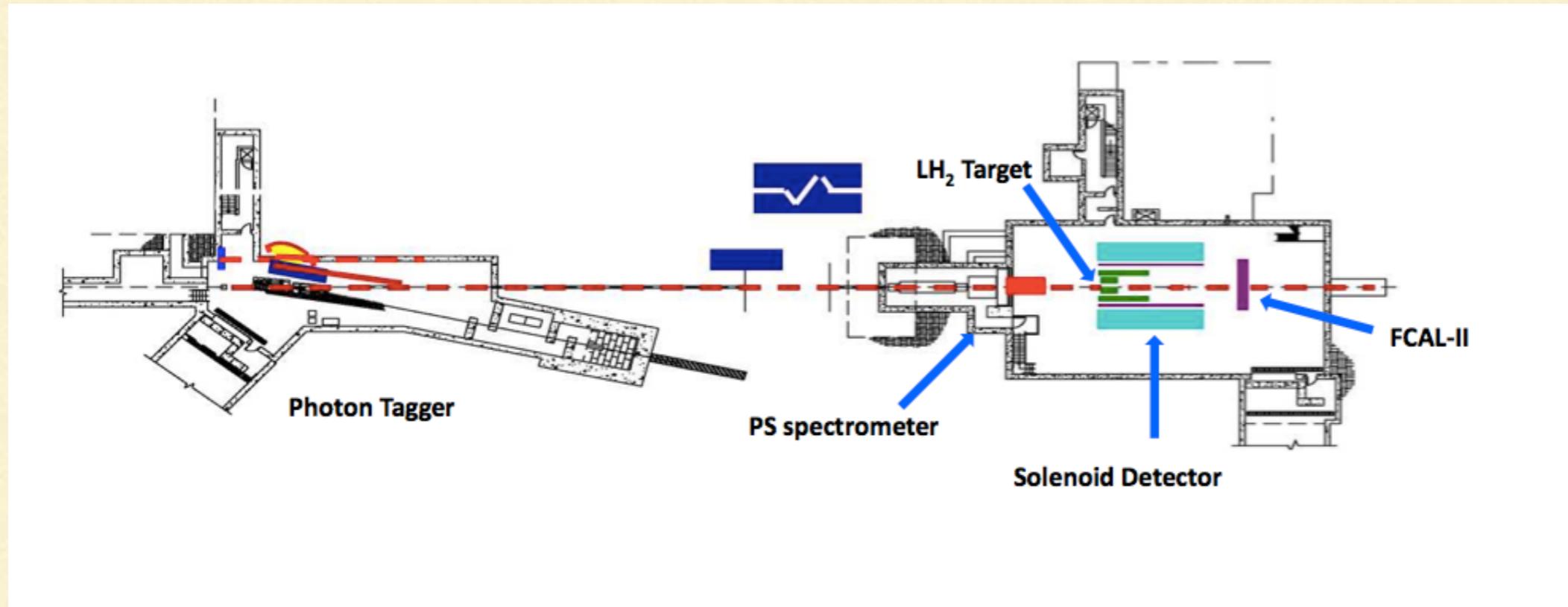
Picture credit: PRIMEX



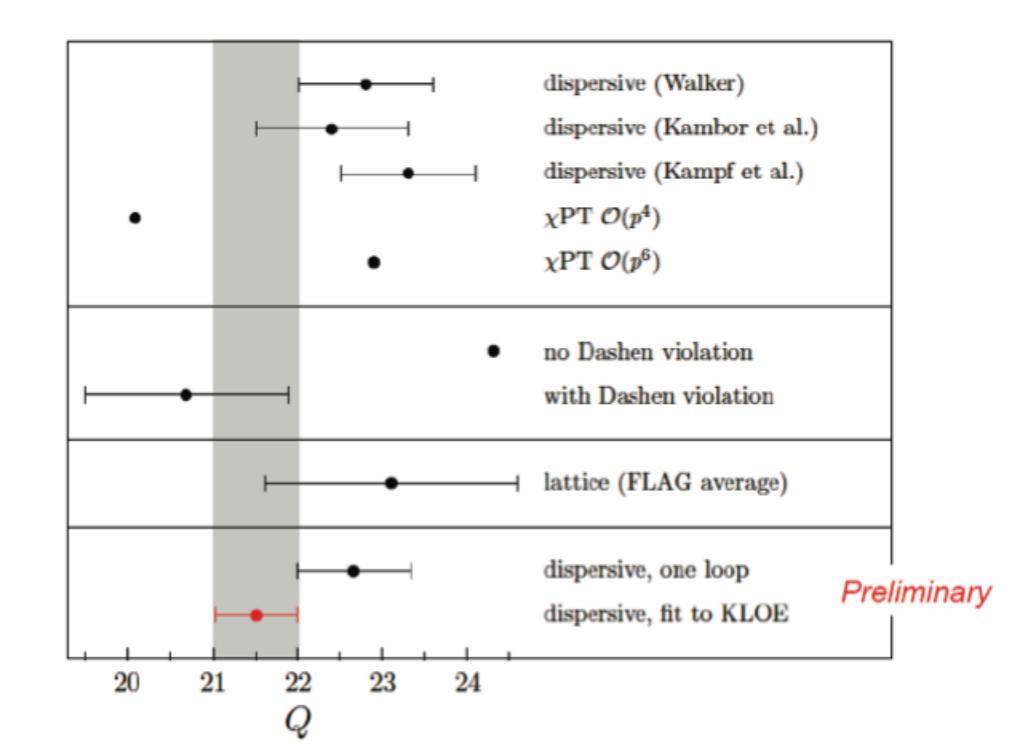
- Fixes normalization of pion transition form-factor: input for hadronic LbL
- χ PT at $\mathcal{O}(p^4)$: $\Gamma(\pi^0 \rightarrow \gamma\gamma) = \frac{\alpha_{\text{em}}^2 N_c^2 m_\pi^3}{576\pi^3 f_\pi^2} = 7.725 \text{ eV}$
- $\mathcal{O}(p^6)$ χ PT: $\Gamma(\pi^0 \rightarrow \gamma\gamma) = (8.10 \pm 0.08) \text{ eV}$
- PRIMEX: $\Gamma(\pi^0 \rightarrow \gamma\gamma) = (7.82 \pm 0.17_{\text{stat.}} \pm 0.16_{\text{syst.}}) \text{ eV}$
- PRIMEX-II to shrink error bar by factor of 2

THE JLAB ETA FACTORY

Picture credits: JEF proposal



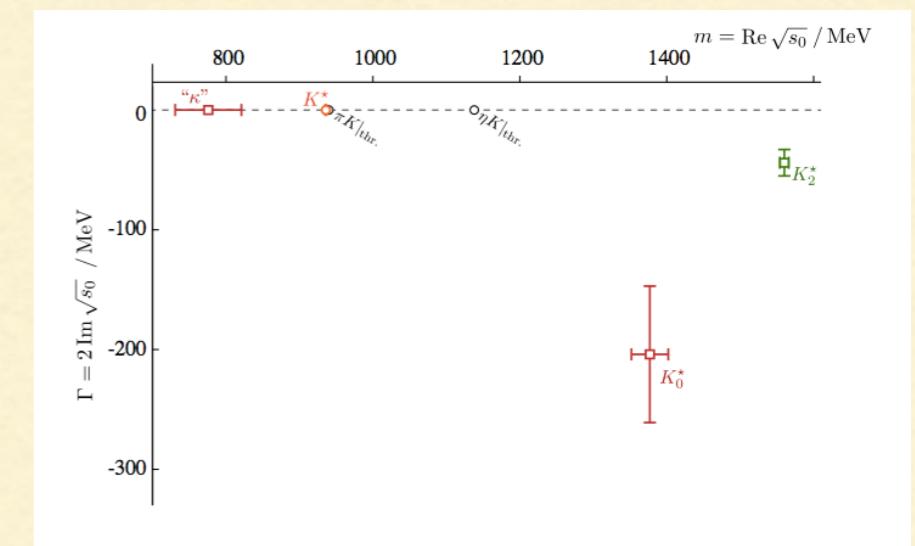
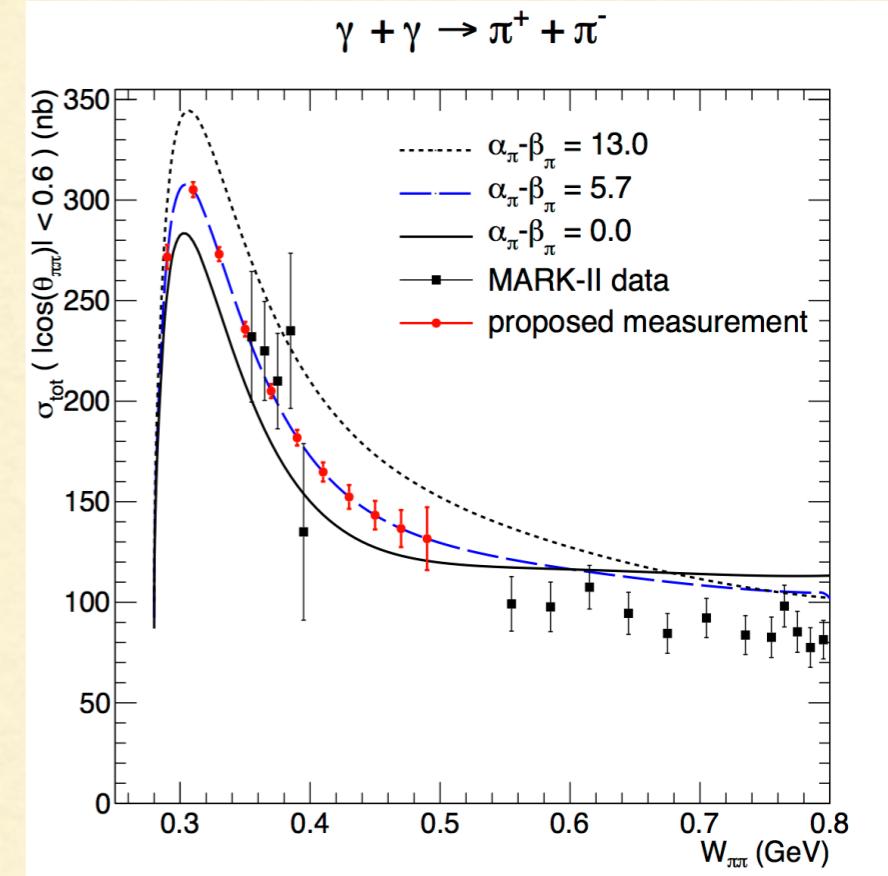
- JEF will improve limits on rare C-violating η decays
- Improved $(m_d - m_u)/m_s$ from $\eta \rightarrow 3\pi$: precise determination of SM parameter
- Go beyond naive χ PT expansion: unitarization.



FUTURE OPPORTUNITIES: A=0

- Two-loop [$\mathcal{O}(P^6)$] χ PT for π properties now standard; high-accuracy calculations routine.
- Convergence in SU(3) not as good. Uncertainty quantification a challenge for initial states with $M \sim 1$ GeV.
- Increasingly precise LQCD numbers for pion dynamics, at lower pion masses
- A=0 resonances from LQCD: ρ , strange states,
- Dispersion relations/unitarized ChiPT/models will connect both experiment and lattice to resonance properties.
- Dispersion relations for HLbL: follow the roadmap
- 12 GeV JLab: PRIMEX-II, π polarizability, η factory

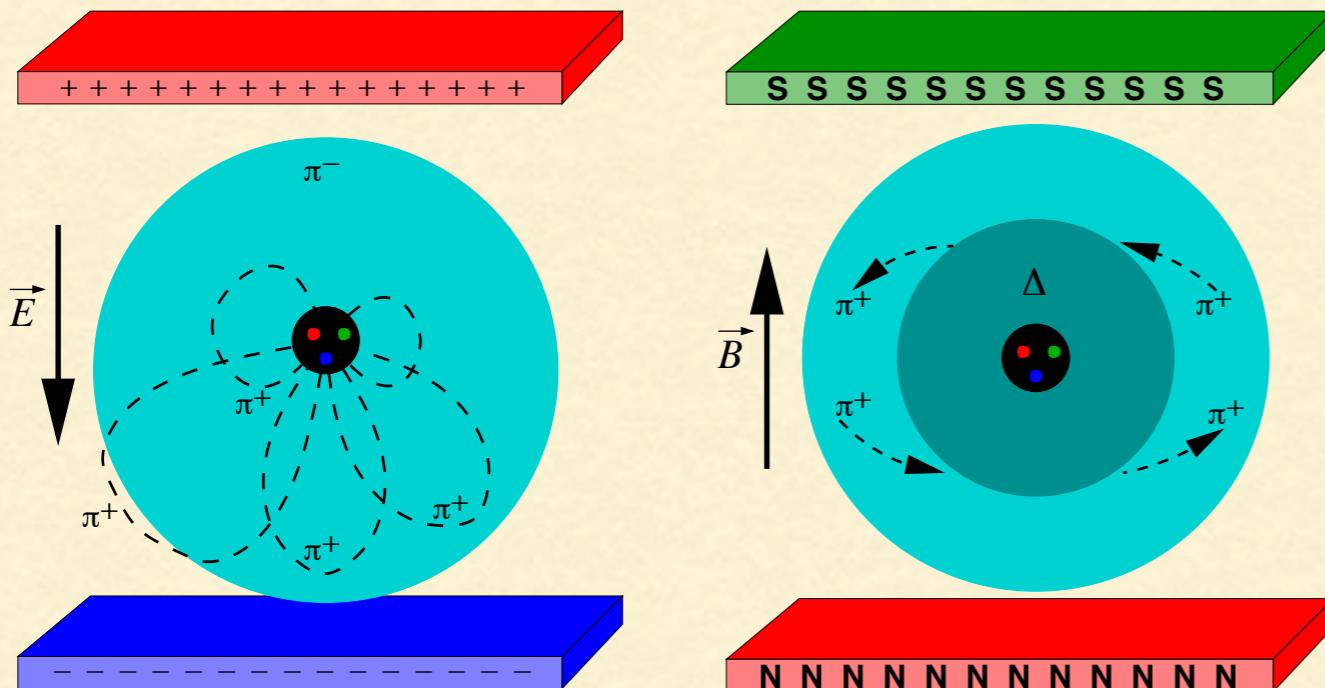
Picture credits: R. Miskimen, J. Dudek et al.



A=1: MOSTLY COMPTON

- Traditionally: “tests of chiral symmetry” in near-threshold pion photoproduction
- Emergent dynamics of strong QCD, but why these matrix elements?
- Here emphasis is on Compton scattering, especially the quest to measure electromagnetic polarizabilities of the proton (neutron in A=2 section)
 - Probe chiral dynamics *below pion threshold*
 - LQCD error bars commensurate with experiment → **TIBURZI**
 - Key inputs in forefront SM parameter extractions and tests: M_n - M_p , μ_H , ...
- Ability to quantify theory uncertainty is key to reliable extraction of hadronic matrix elements

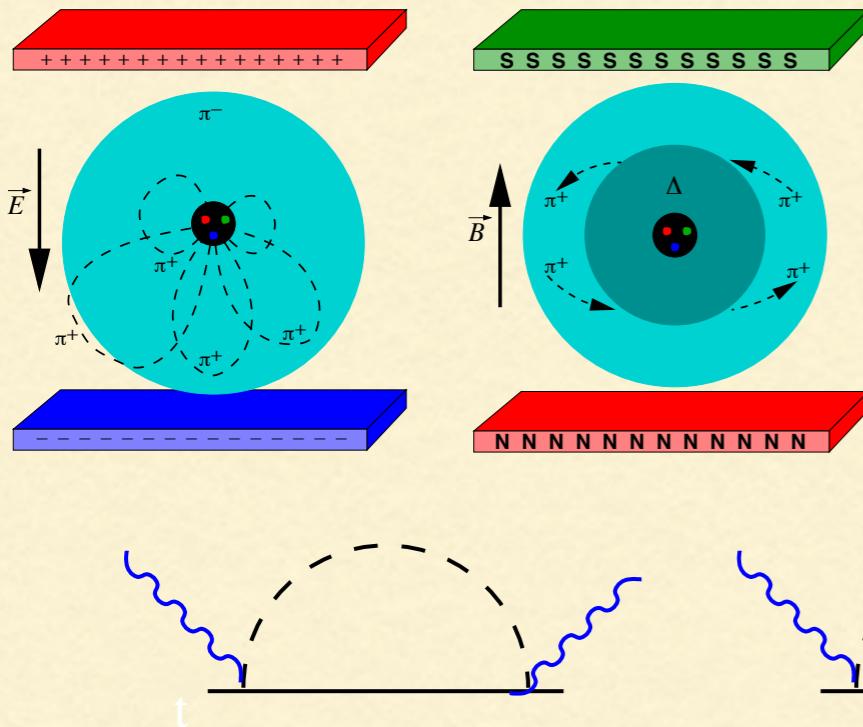
STATIC AND DYNAMIC POLARIZABILITIES



Picture credit: H. Grießhammer

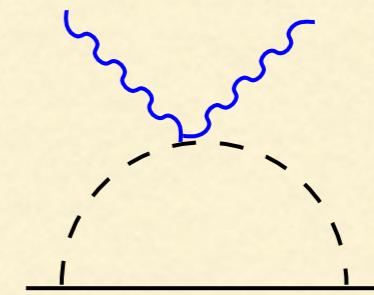
$$H = -2\pi[\alpha_{E1}\mathbf{E}^2 + \beta_{M1}\mathbf{B}^2 + \dots]$$

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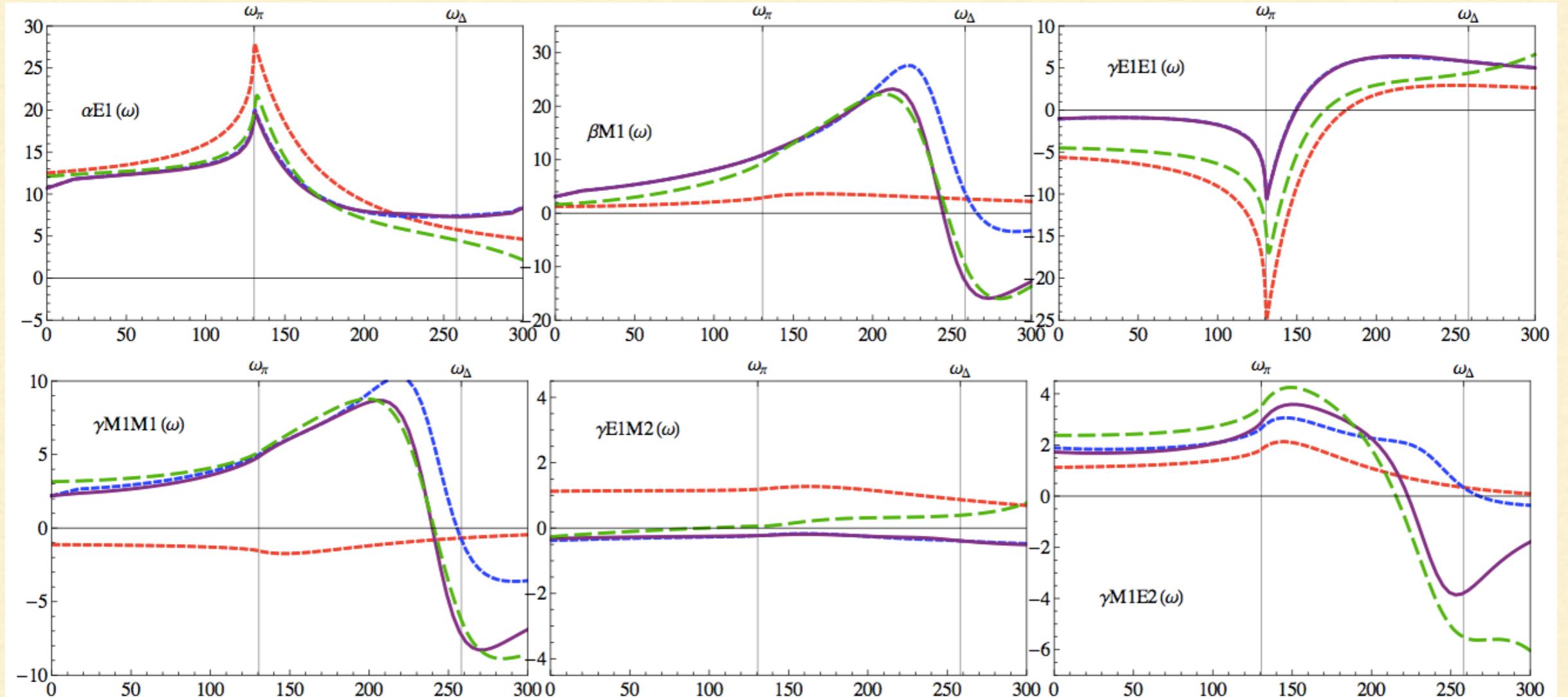


- Δ -less χ PT at $\mathcal{O}(P^3)$: $\alpha_{E1}^{(p)} = 10\beta_{M1}^{(p)} = \frac{5e^2 g_A^2}{384 f_\pi^2 m_\pi} = 12.5 \times 10^{-4} \text{ fm}^3$
- But χ PT predicts entire energy dependence of amplitudes, so define:

$$H = -2\pi[\alpha_{E1}(\omega)\mathbf{E}^2 + \beta_{M1}(\omega)\mathbf{B}^2 + \gamma_{E1E1}(\omega)\sigma \cdot (\mathbf{E} \times \dot{\mathbf{E}}) + \gamma_{M1M1}(\omega)\sigma \cdot (\mathbf{B} \times \dot{\mathbf{B}}) - 2\gamma_{M1E2}(\omega)\sigma_i B_j E_{ij} + 2\gamma_{E1M2}(\omega)\sigma_i E_j B_{ij} + \dots]$$
- Numerically: six dynamical polarizabilities reproduce Compton observables up to 300 MeV to a few per cent: partial-wave analysis of γp
(Was made rigorous via multipole expansion of different invariant amplitudes.)

DYNAMICAL POLARIZABILITIES

Grießhammer, McGovern, Phillips, Feldman



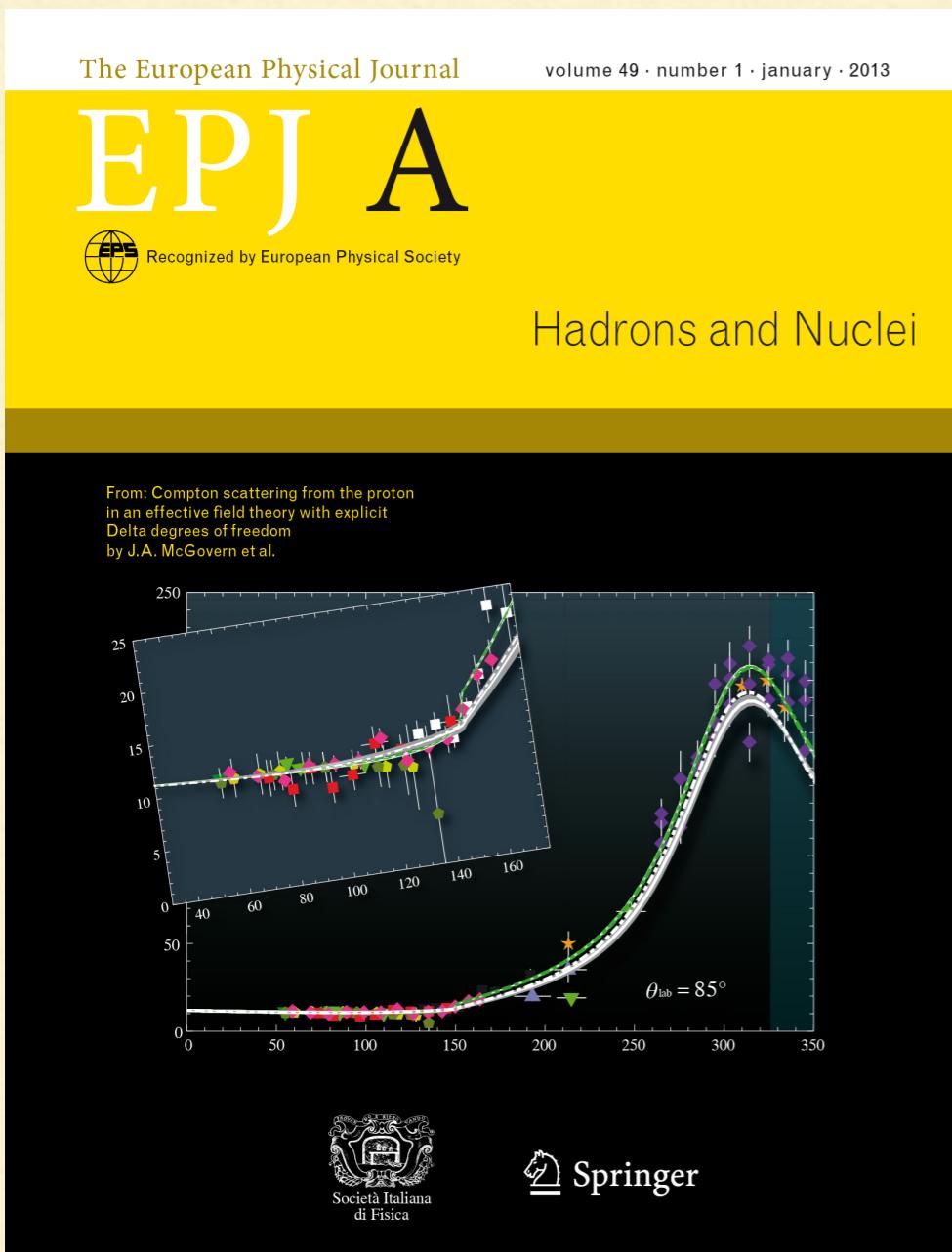
- Remarkable concordance of various theories on energy dependence of dynamical polarizabilities below 200 MeV.

Grießhammer, L'vov, McGovern, Pascalutsa,
Pasquini, Phillips, arXiv:1409.1512

- Suggests: (a) experiments should be done in this regime in order to minimize theory uncertainties; (b) best to fit all six static pols to experiment.

NEW ANALYSIS OF γ -PROTON DATA

- Ingredients: χ PT $N\pi$ loops up to $O(P^4)$ + Dynamical Δ + $\Delta\pi$ loops + Δ width and dressed $\gamma N \Delta$ vertex [up to $O(e^2 \delta^4)$]: three free parameters



- Good reproduction of data up to ≈ 350 MeV
- Develop world database up to 200 MeV (incl. treatment of systematic errors). Greater theory accuracy compared to Δ region (N^4LO vs NLO).
- Fit to world γp data up to 170 MeV:
 $\chi^2/\text{d.o.f.} = 110.5/134$
 $\alpha_{E1}^{(p)} = 10.65 \pm 0.35(\text{stat}) \pm 0.2(\text{Baldin}) \pm 0.3(\text{theory})$
 $\beta_{M1}^{(p)} = 3.15 \mp 0.35(\text{stat}) \pm 0.2(\text{Baldin}) \mp 0.3(\text{theory})$
McGovern, Grießhammer, Phillips (2013)
- $\beta_{M1}^{(p)}$ markedly higher than in previous (DR) fits (although marginally consistent within error bars)

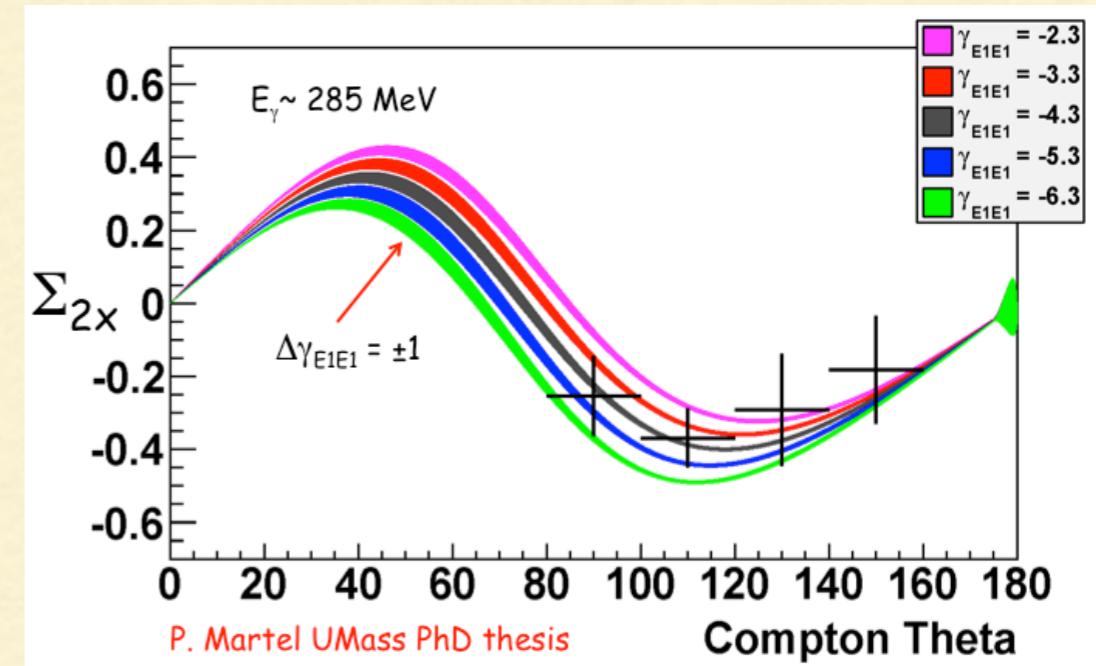
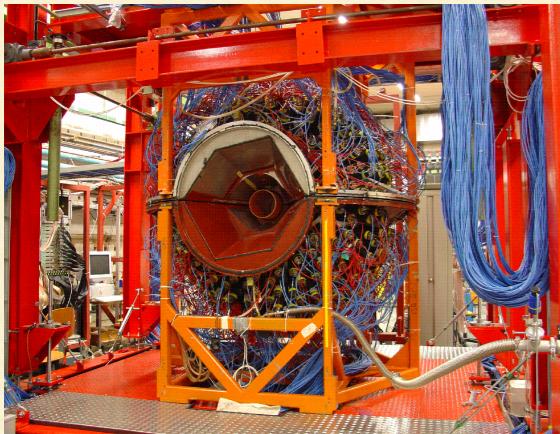
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- Response of degrees of freedom that carry nucleon spin to E and B fields

- LQCD results in progress → **TIBURZI**



| | χ EFT | Fixed-t DR | Experiment |
|-------|-------------|------------|------------|
| E1 E1 | -1.1±1.8 | -3.85±0.45 | -3.5±0.12 |
| M1 M1 | 2.2±0.5±0.7 | 2.8±0.1 | 3.16±0.85 |
| E1 M2 | -0.4±0.4 | -0.15±0.15 | -0.7±1.2 |
| E2 M1 | 1.9±0.4 | 2.0±0.1 | 1.99±0.29 |

Spin polarizabilities, all in units of 10^{-4} fm⁴

Future at MAMI:
measurements of Σ_{2z} , Σ_3 .
Full multipole analysis of
suite of Compton
observables

Future at HIGS: low-energy

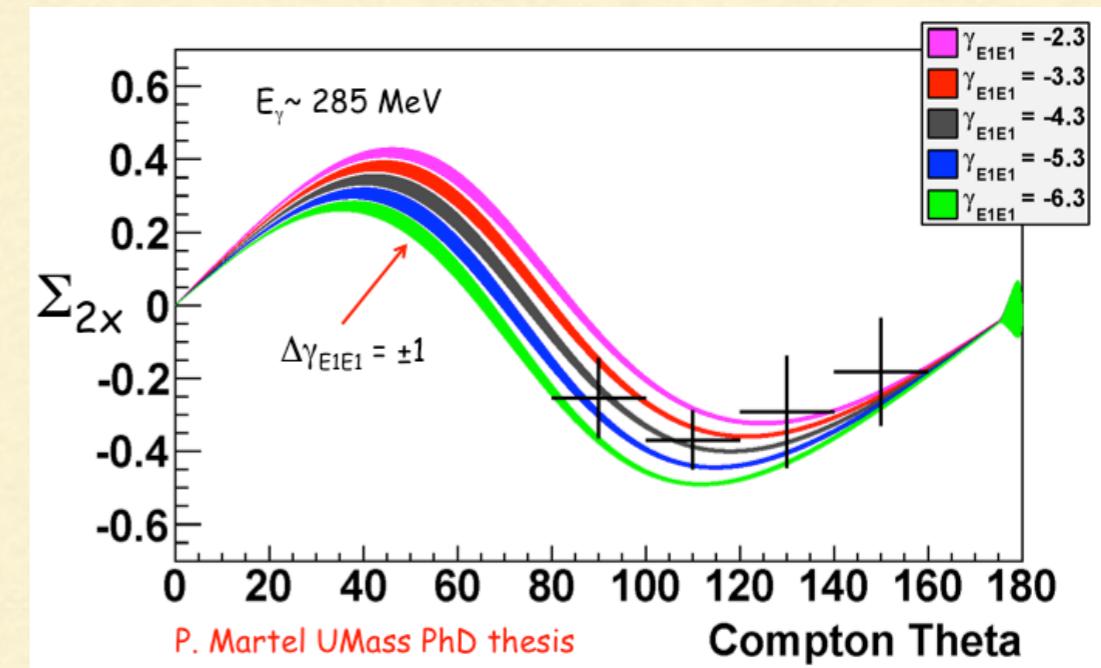
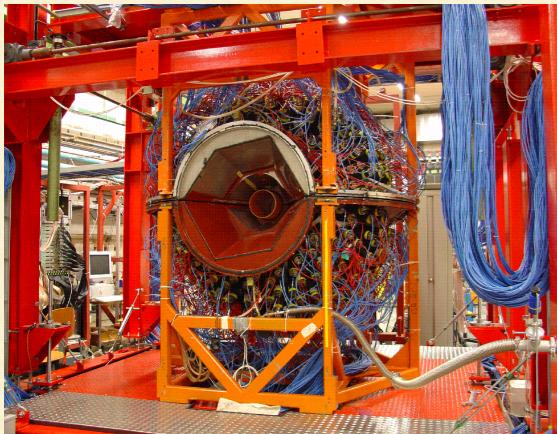
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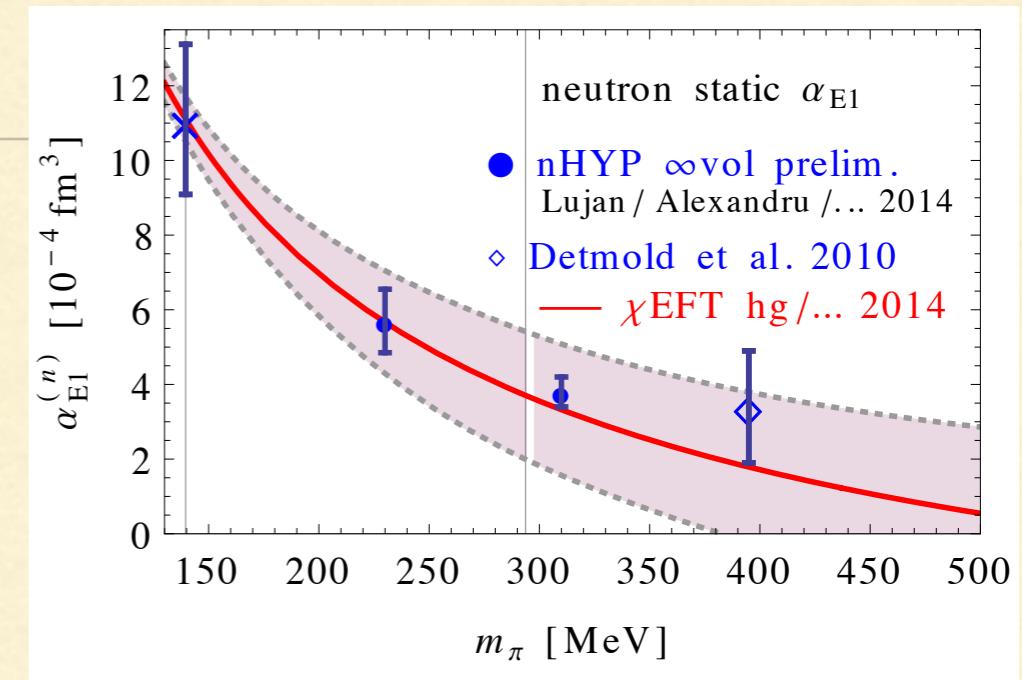
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NEXT TALK

FUTURE: A=1

Picture credits: A. Alexandru et al., Chiral MAID



MAMI, HIGS

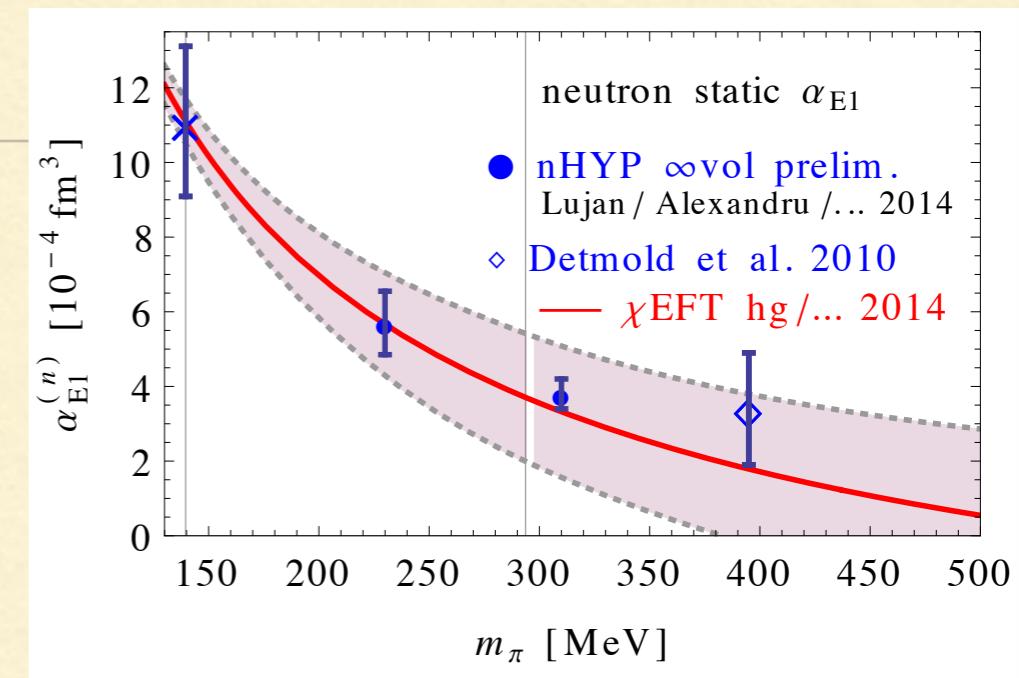
Input for polarizability correction to extraction of r_p from μH

FUTURE: A=1

Picture credits: A. Alexandru et al., Chiral MAID

- Compton:

- Lattice results: α , β , γ s
- First asymmetry data below 200 MeV
- High-accuracy extraction of γ 's
- Resolution of high/low $\beta_{\text{MI}}^{(P)}$ issue via precision experiment and theory
Input for polarizability correction to extraction of r_p from μH
- Analyses of higher-energy data: combination of χ PT and DR. Multipole analysis
Role of 0^{++} exchange in t-channel
- VCS is a largely unexplored frontier → probe spatial distribution of α , β , γ s



MAMI, HIGS

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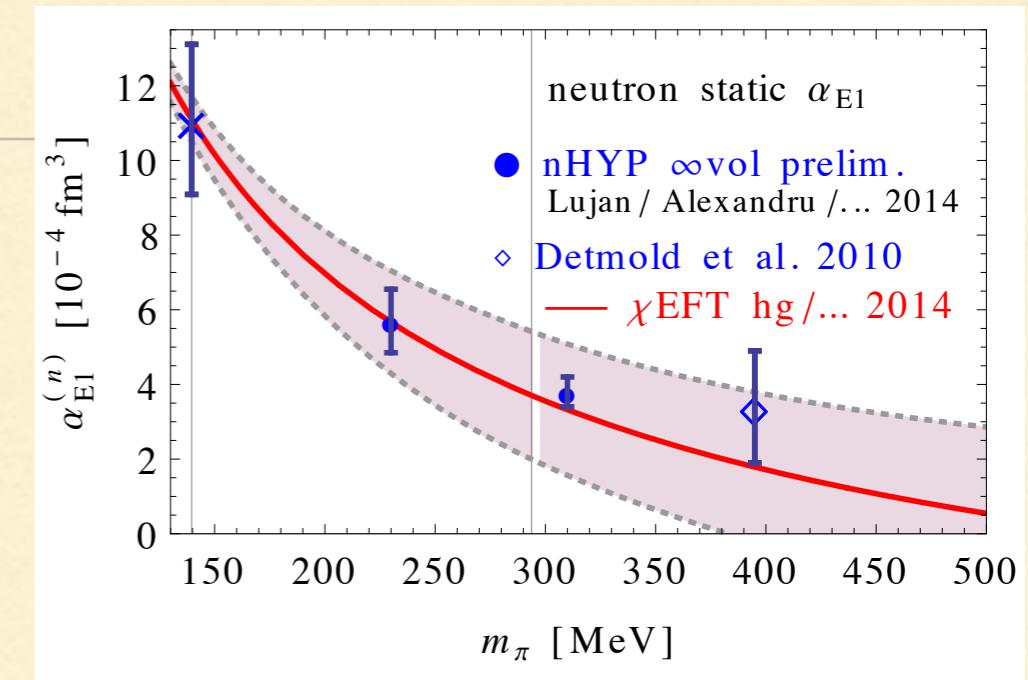
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■ Pion photo- and electro-production

- New results from MAMI (US involvement!); analysis of Hall A data in progress
- Chiral MAID: need to add $\Delta(1232)$ d.o.f.



MAMI, HIGS

FUTURE: A=1

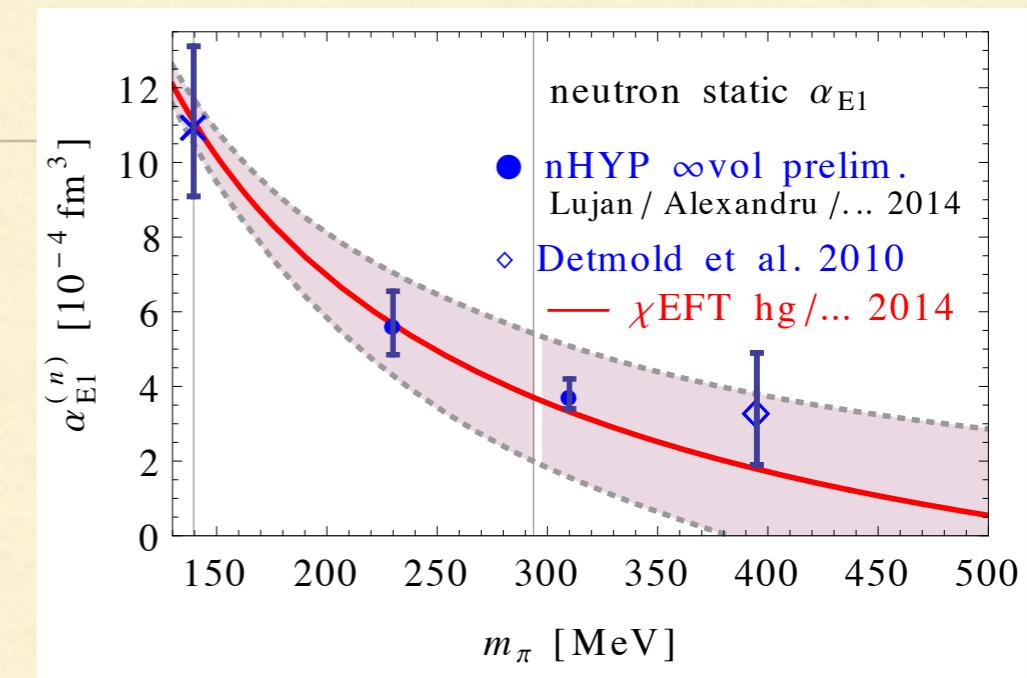
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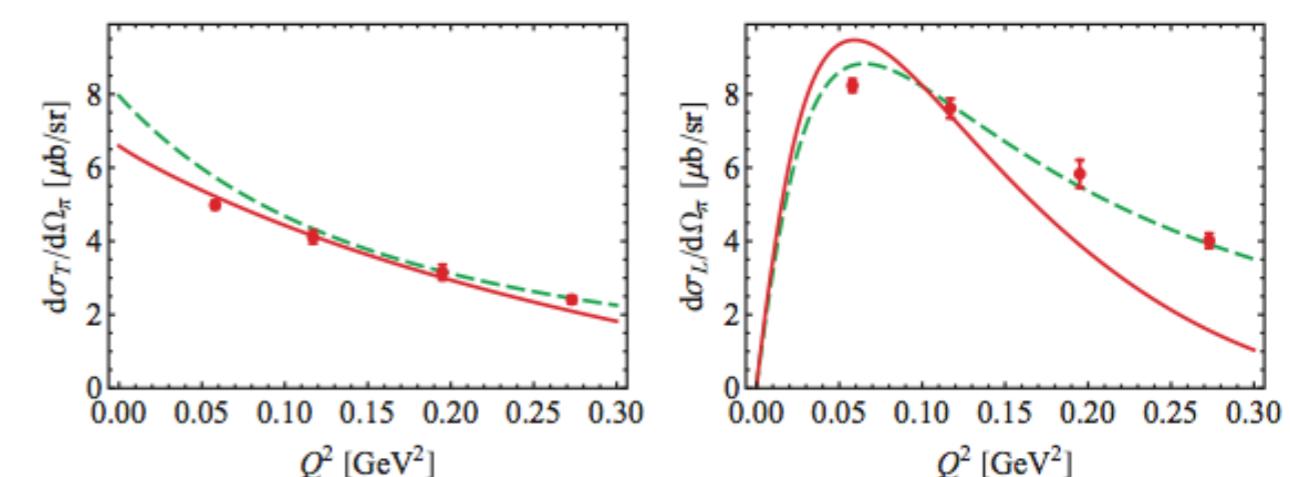
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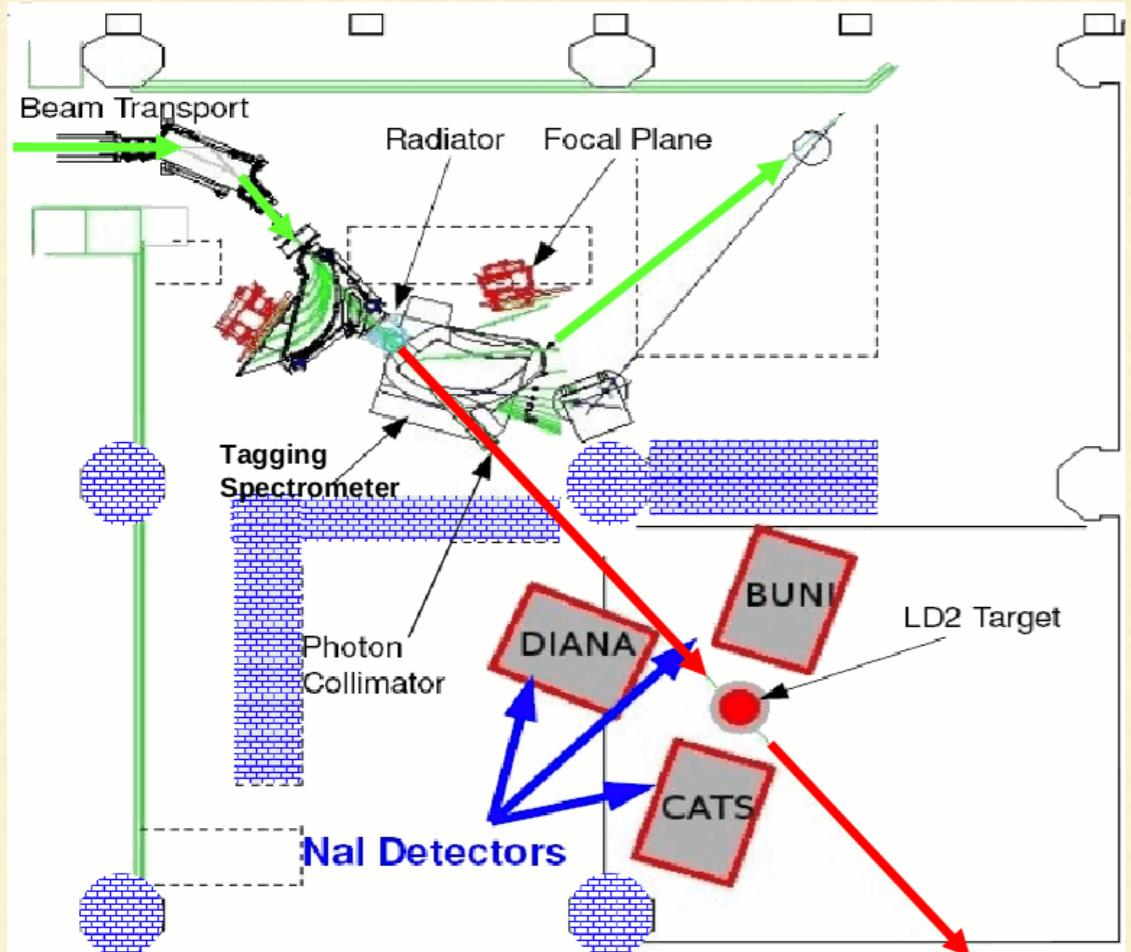
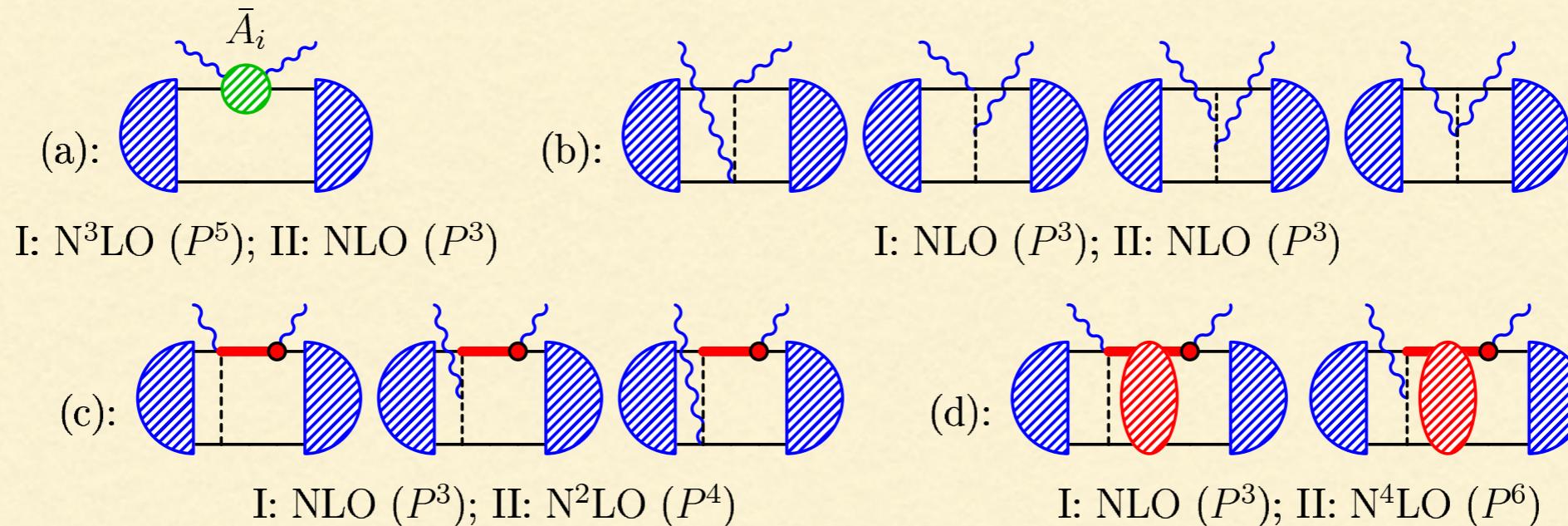


MAMI, HIGS



A=2: GETTING AT THE NEUTRON

Picture credits: H. Grießhammer, L. Myers

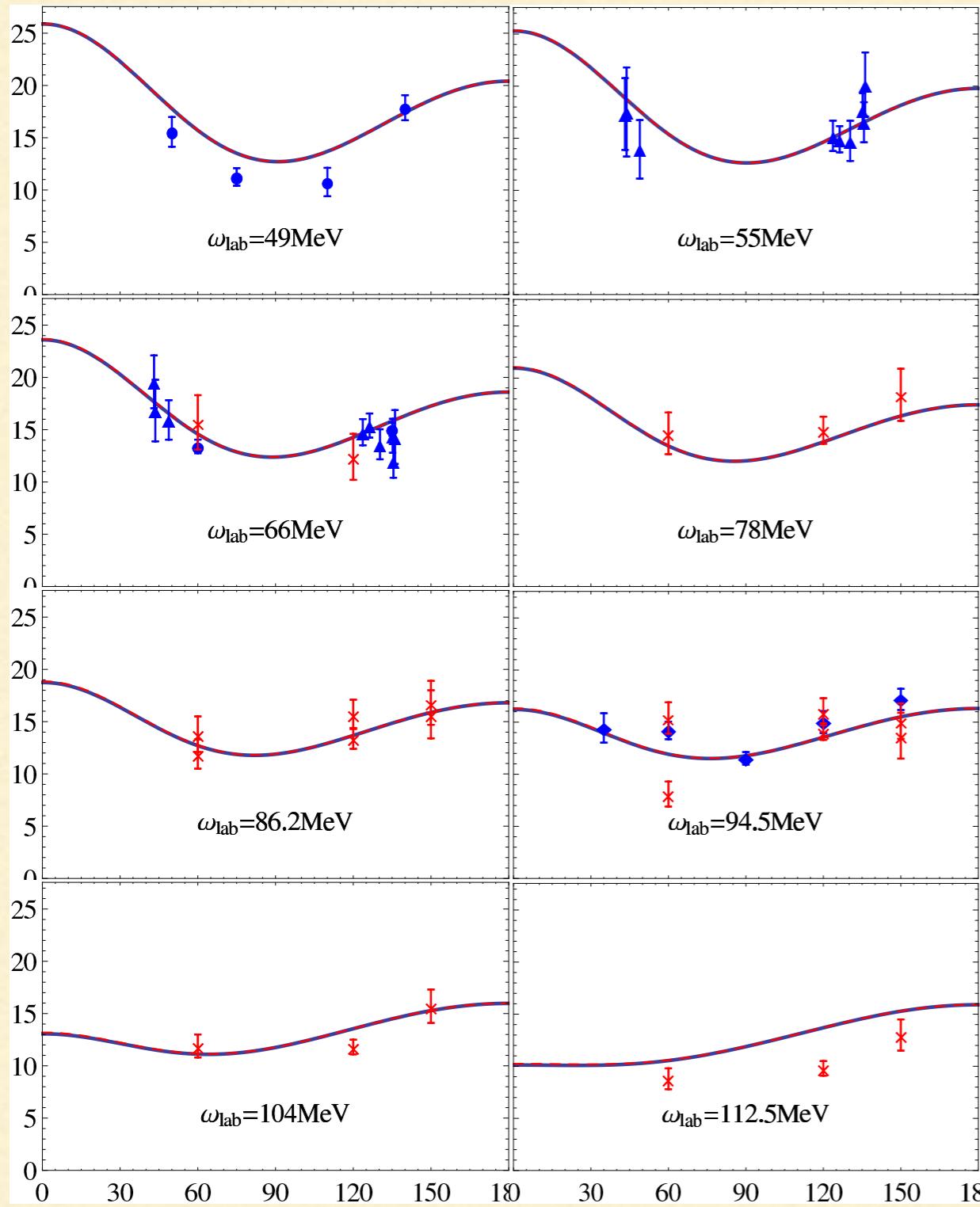


- Extract isoscalar combination of nucleon polarizabilities from elastic γd
- COMPTON@MAX-Lab collaboration: unification of γd expertise. Theory support.
- 15 years of development in χ EFT: reached accuracy at or beyond experiment

→ GRIESSHAMMER

COMPTON@MAXLAB

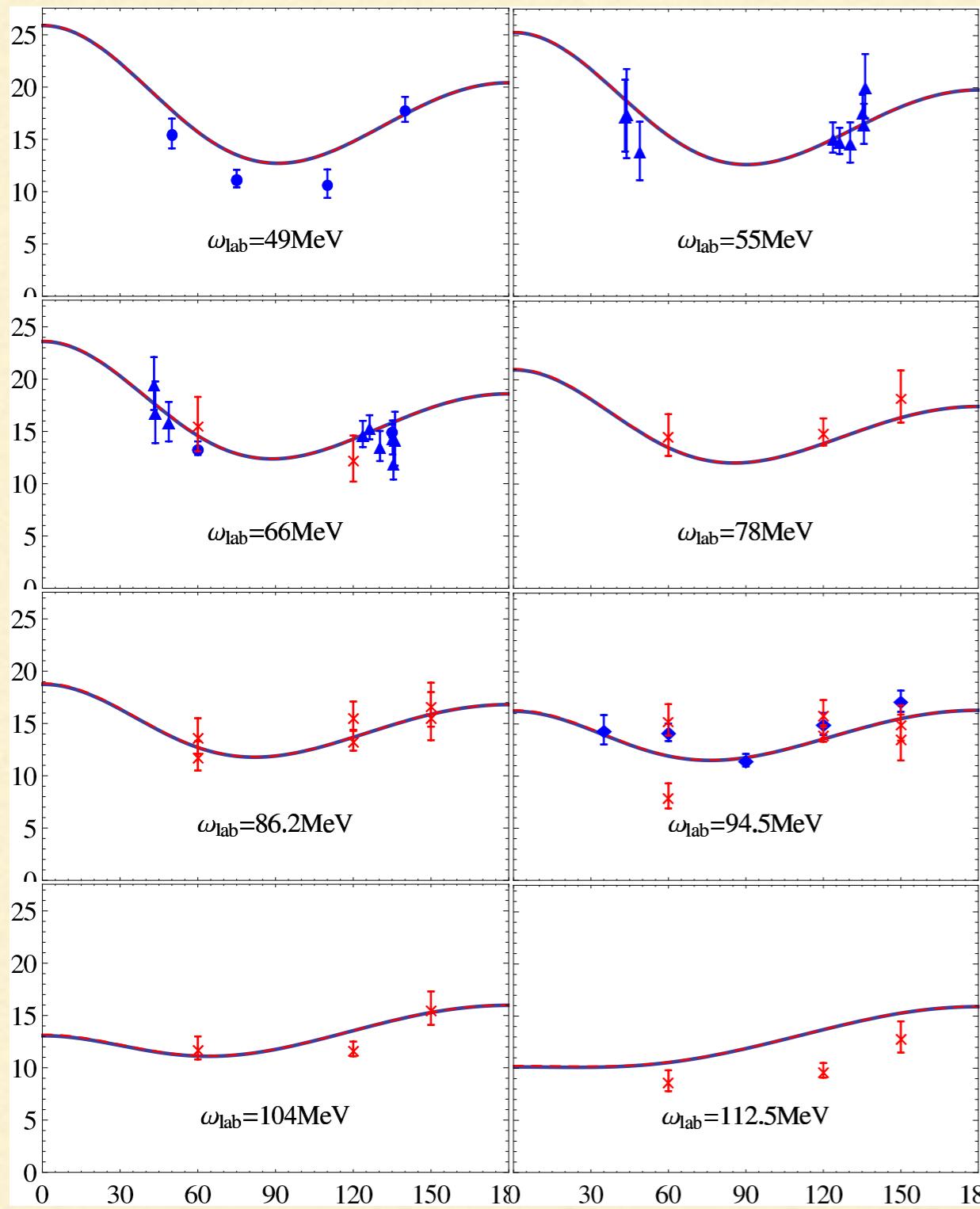
Picture credit: arXiv:1409.????



- U.S.-U.K.-Sweden collaboration
- Doubled world data-base
- Hint of $\beta_n > \beta_p$? $\beta_n - \beta_p$ presently most uncertain input to $(M_p - M_n)^{\text{em}}$
- Higher-energy data being analyzed; theory for $E_\gamma > m_\pi$ under development
- Consistent with quasi-free $\gamma d \rightarrow \gamma np$
- Future QF: JLAB FEL for ARCS? Theory.
- Future: precision at lower energies; asymmetries \rightarrow neutron γ 's

COMPTON@MAXLAB

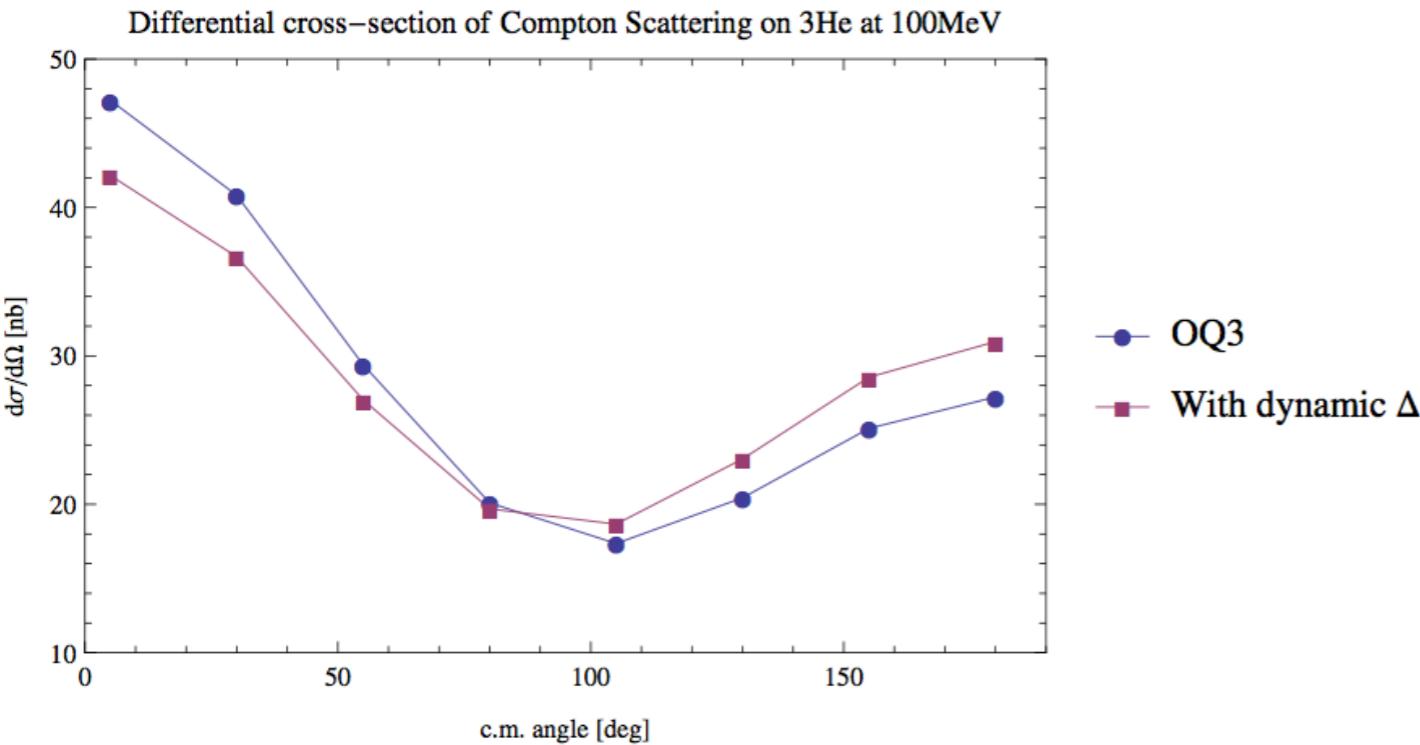
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- U.S.-U.K.-Sweden collaboration
- Doubled world data-base
- $\alpha_n = 11.65 \pm 1.25(\text{stat}) \pm 0.2(\text{BSR}) \pm 0.8(\text{th})$
- $\beta_n = 3.55 \mp 1.25(\text{stat}) \pm 0.2(\text{BSR}) \mp 0.8(\text{th})$
- Hint of $\beta_n > \beta_p$? $\beta_n - \beta_p$ presently most uncertain input to $(M_p - M_n)^{\text{em}}$
- Higher-energy data being analyzed; theory for $E_\gamma > m_\pi$ under development
- Consistent with quasi-free $\gamma d \rightarrow \gamma np$
- Future QF: JLAB FEL for ARCS? Theory.
- Future: Precision at lower energies; asymmetries \rightarrow neutron γs

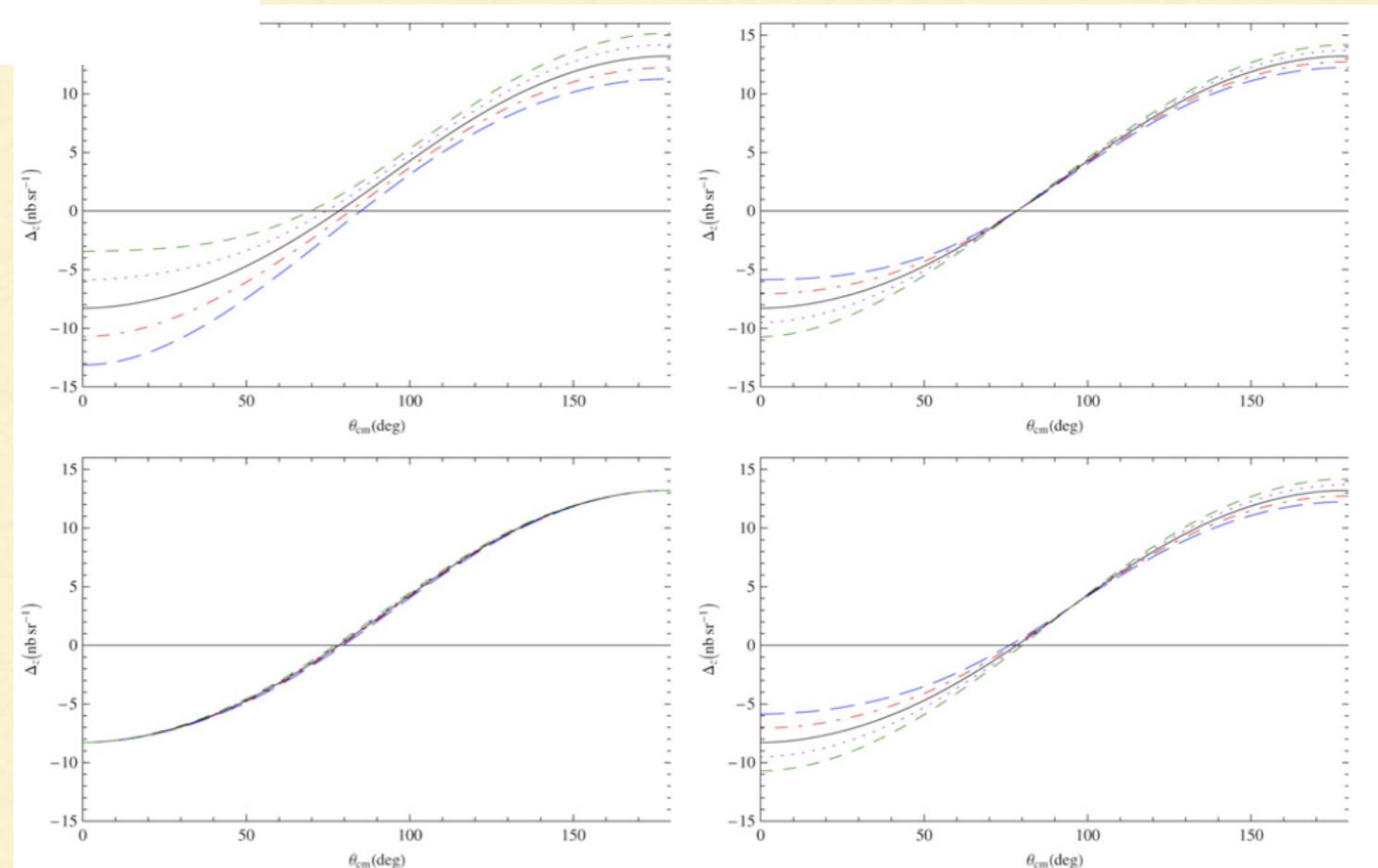
NEXT TALK

COMPTON IN A=3

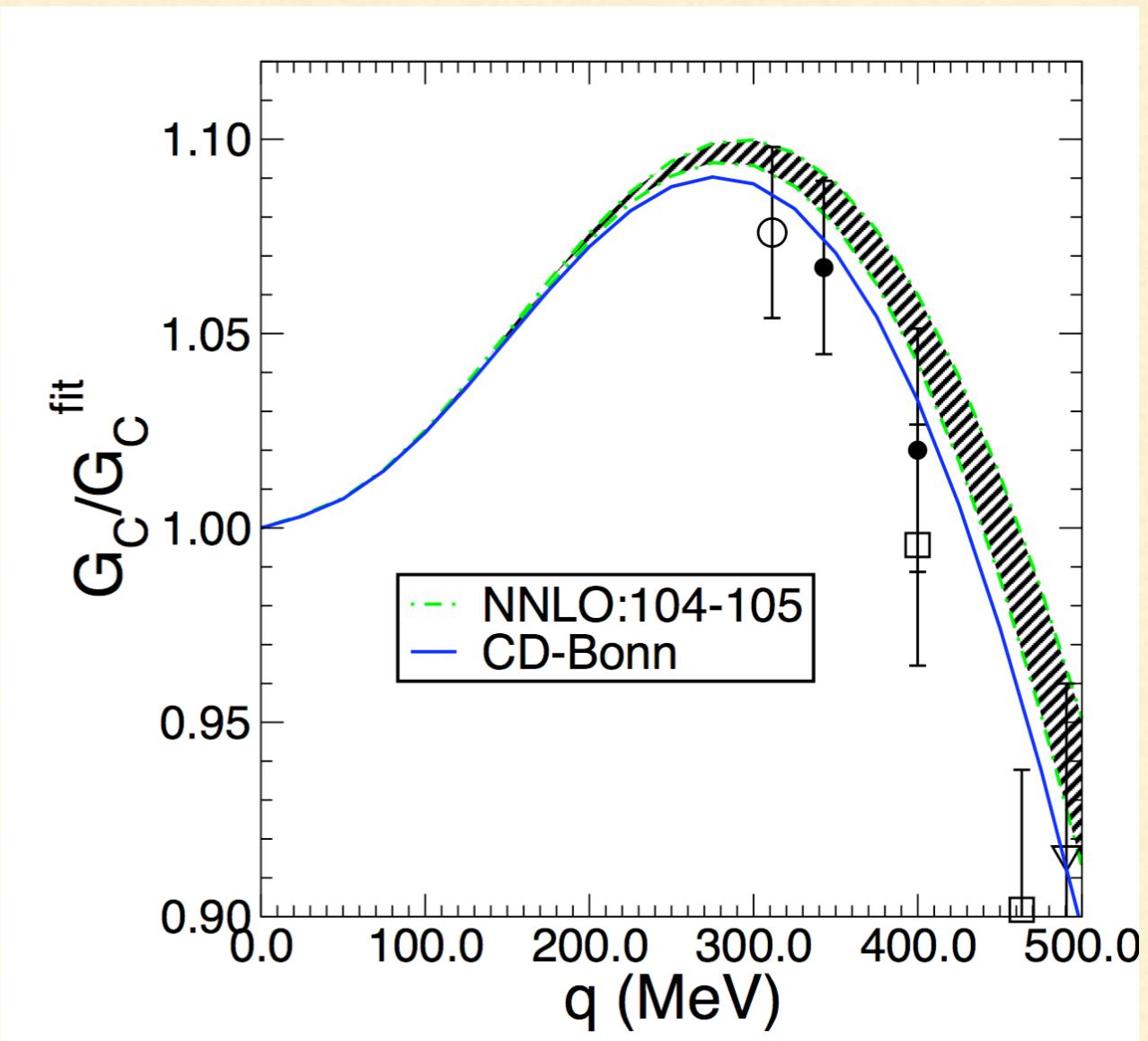
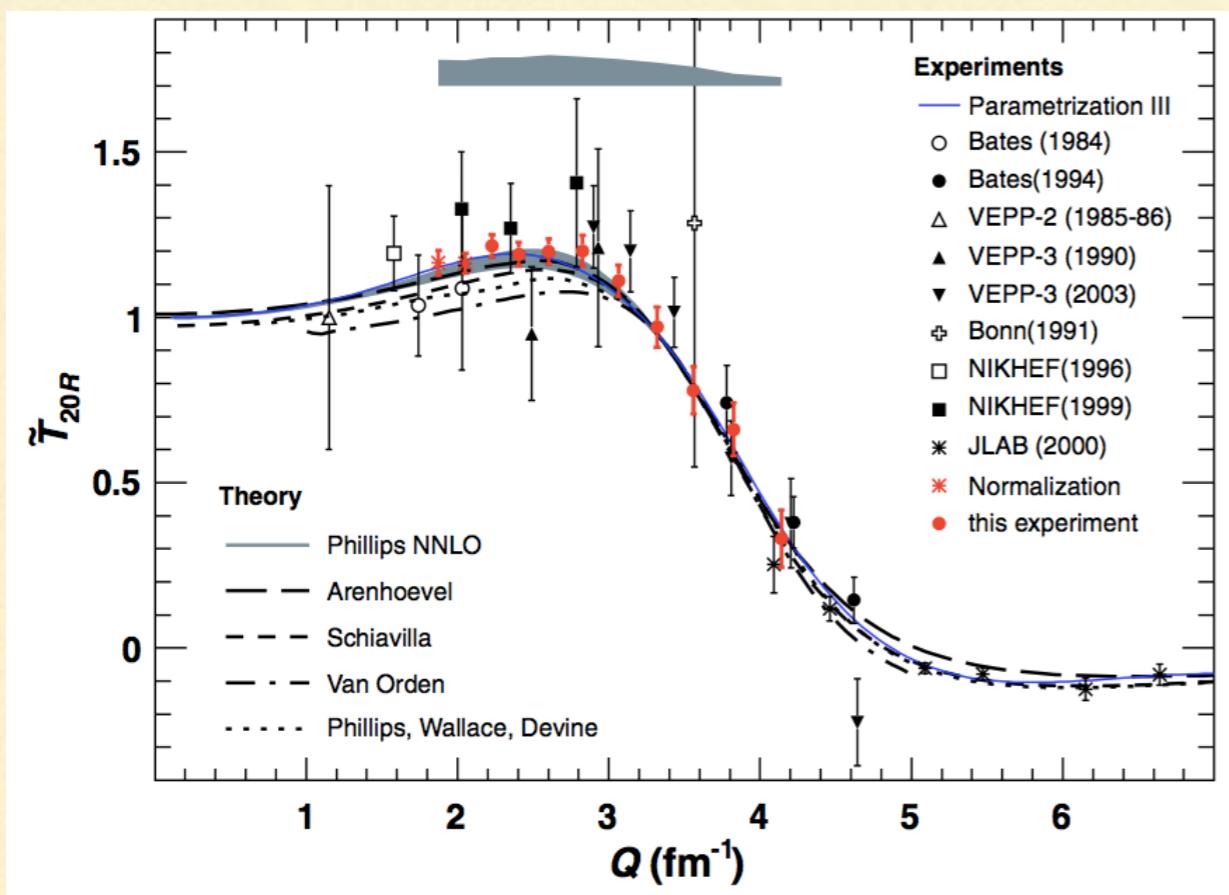


- Recent experimental interest in elastic Compton from ^3He , ^4He
- Larger charge \Rightarrow larger absolute effect of polarizabilities
- Theory: extension of γd

- HIGS: polarized ^3He target
- Asymmetries from polarized ^3He predicted to be as those from a free (polarized) neutron
- Access to neutron γs !



PRECISE EW STRUCTURE IN A=2



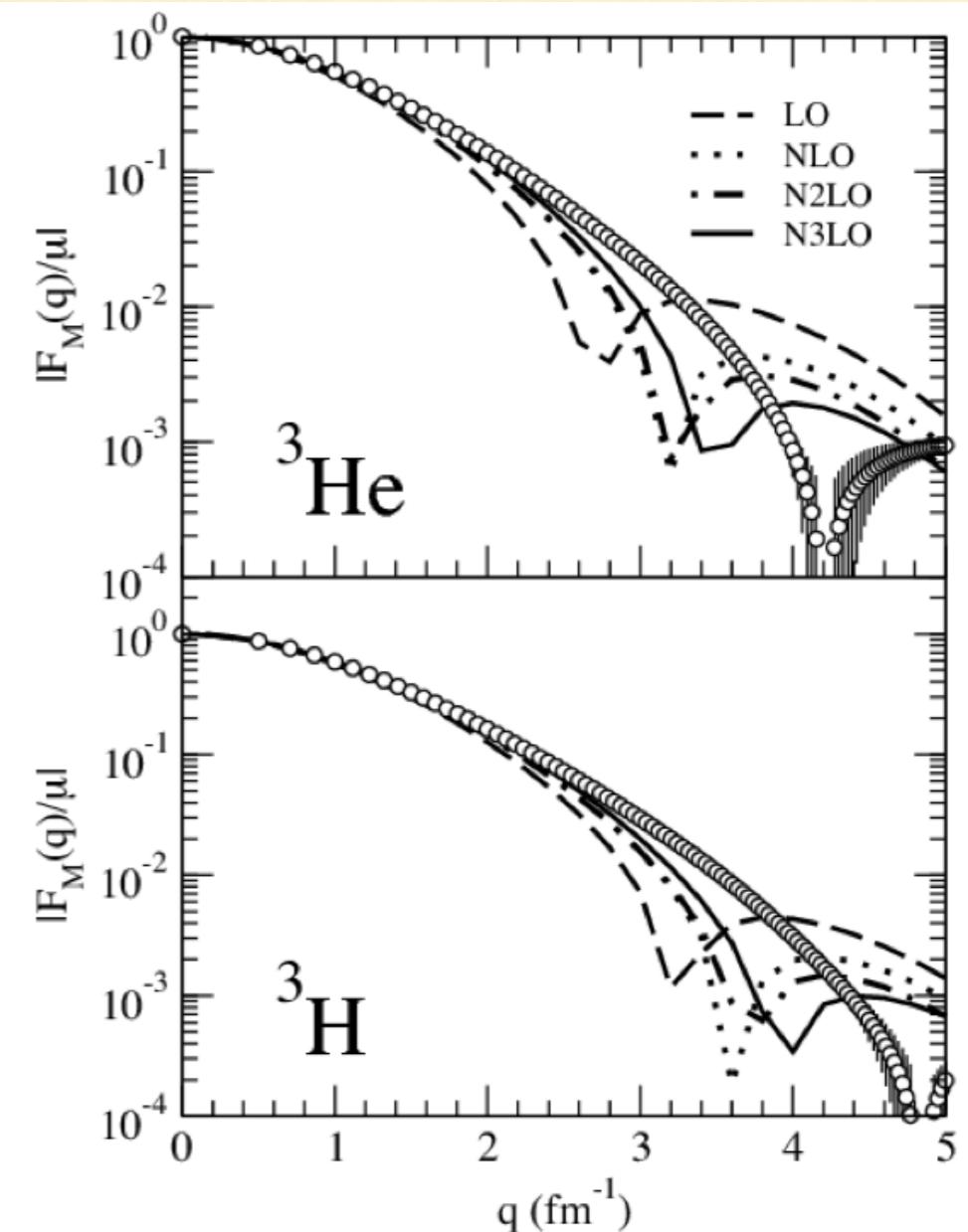
- Constrained to radius: 0.6% error bar in G_C/G_C^{fit} at $Q=0.5 \text{ GeV}/c$
- Precision computation of polarizability corrections in μd : input to BSM search

$$\delta_{2\gamma E} = \delta_{\text{pol}}^A + \delta_{\text{Zem}} + \delta_{\text{pol}}^N = 1.690 \pm 0.020 \text{ meV}$$

- Prediction for MuSun experiment: $\mu^- + d \rightarrow \nu_\mu + n + n; \Gamma_D = 393(3) \text{ s}^{-1}$

BEYOND A=3 AND THE FUTURE

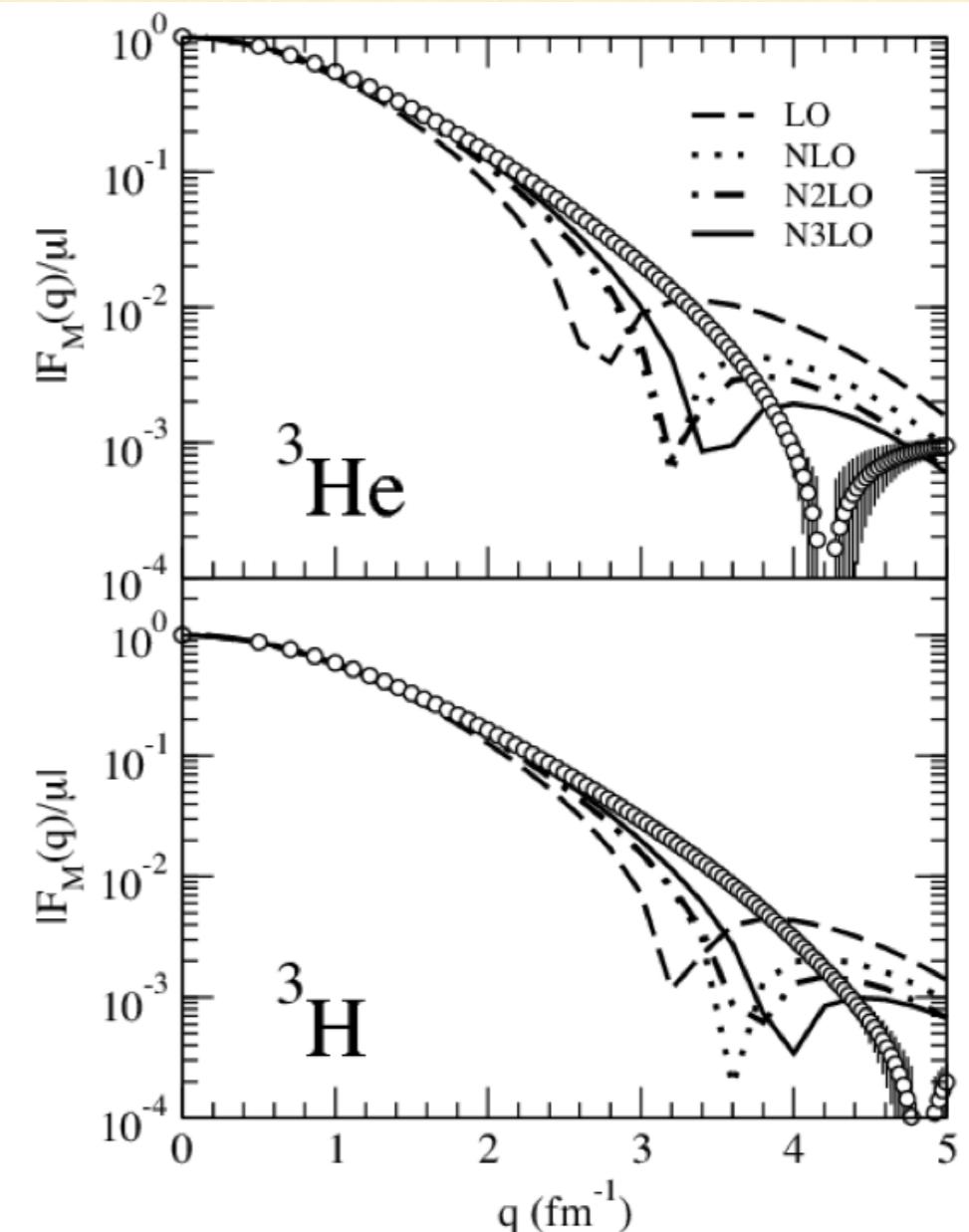
Picture credits: M. Piarulli et al., L. Myers et al., NPLQCD



BEYOND A=3 AND THE FUTURE

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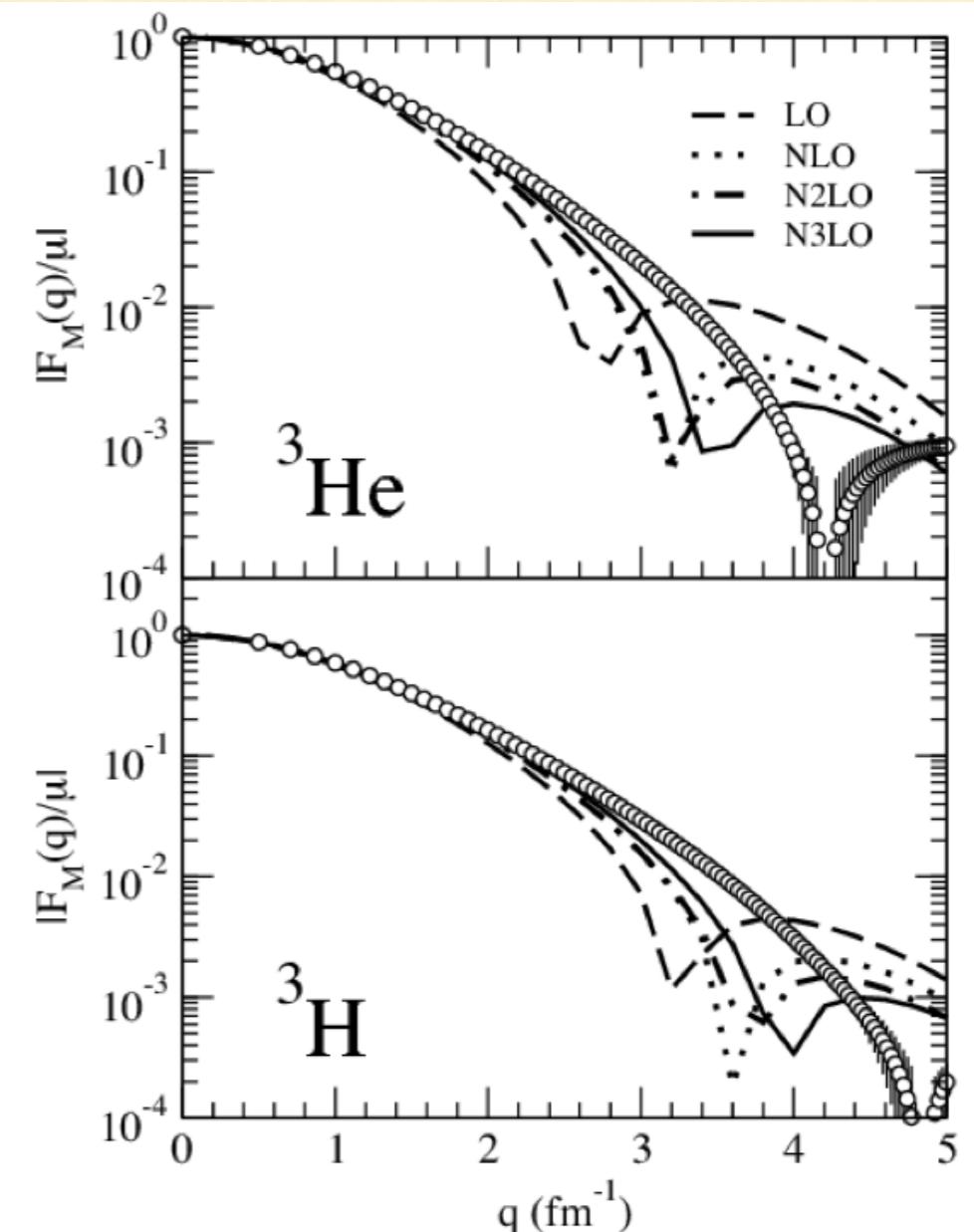
- Analogous computation for μ^- capture on ^3He sets limits on BSM physics
- Confidence in ability of χ EFT to describe few-body electroweak structure



BEYOND A=3 AND THE FUTURE

Picture credits: M. Piarulli et al., L. Myers et al., NPLQCD

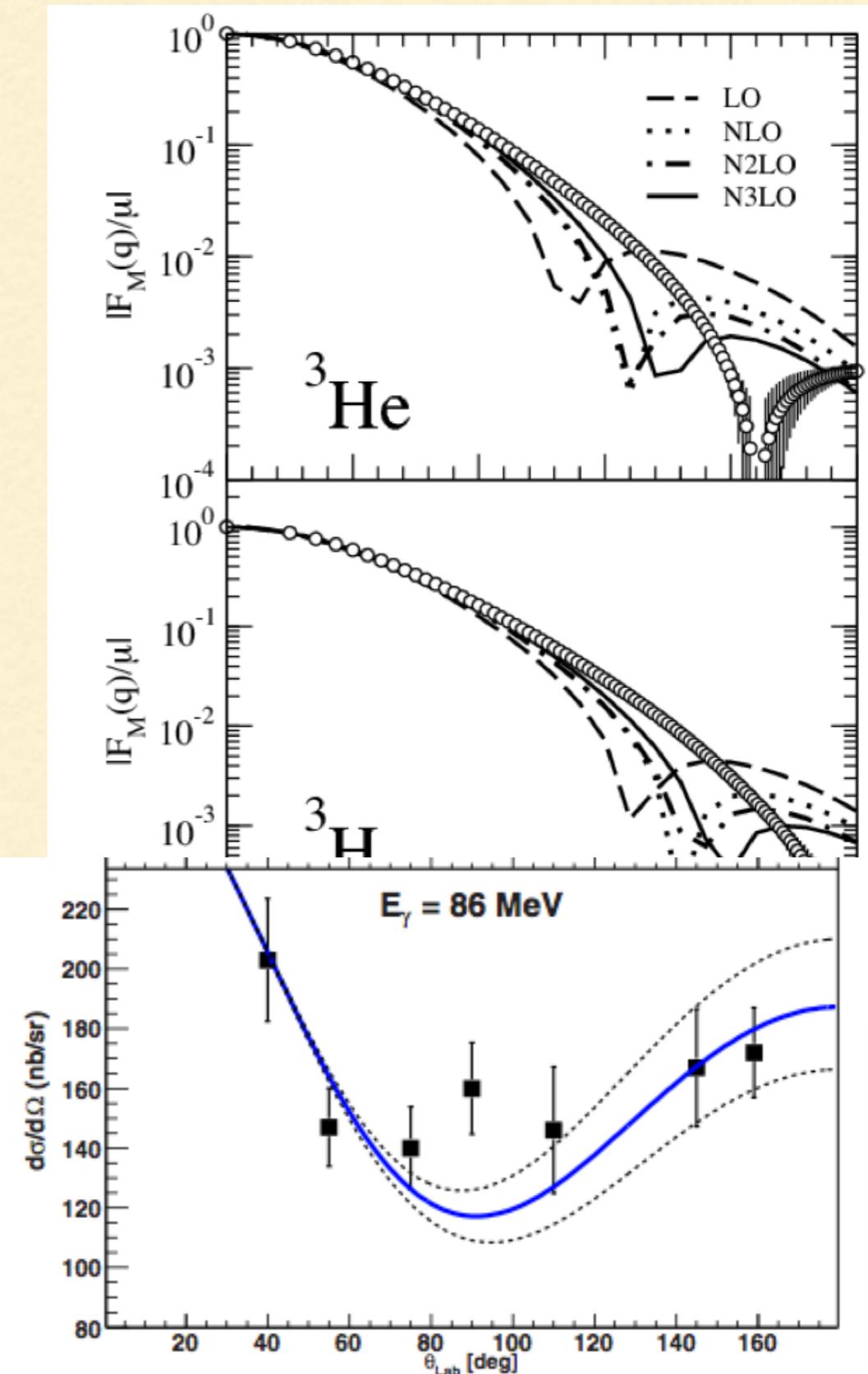
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- Next seven years will see precise calculations of EM structure to at least A=12



BEYOND A=3 AND THE FUTURE

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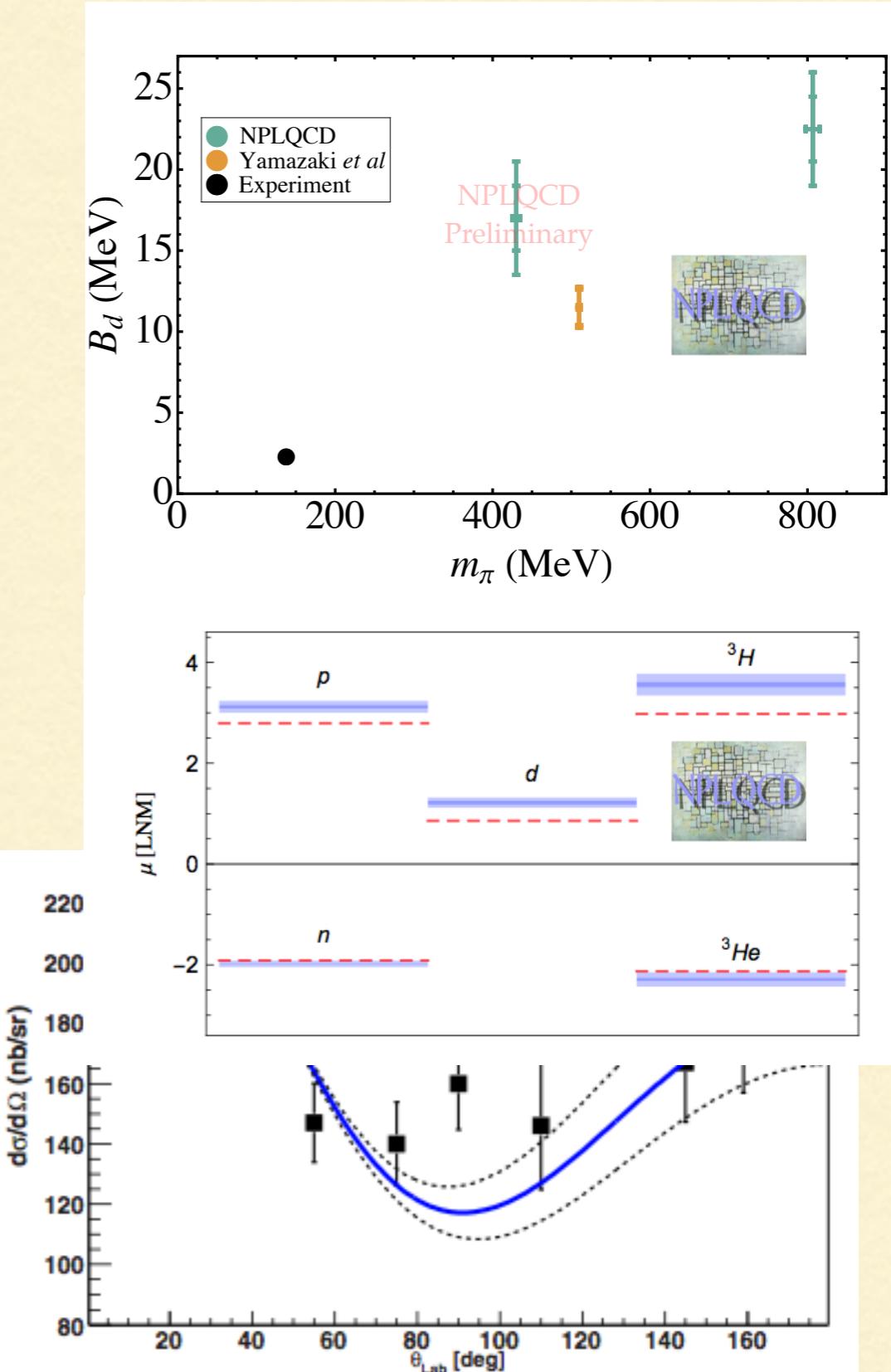


BEYOND A=3 AND THE FUTURE

Picture credits: M. Piarulli et al., L. Myers et al., NPLQCD

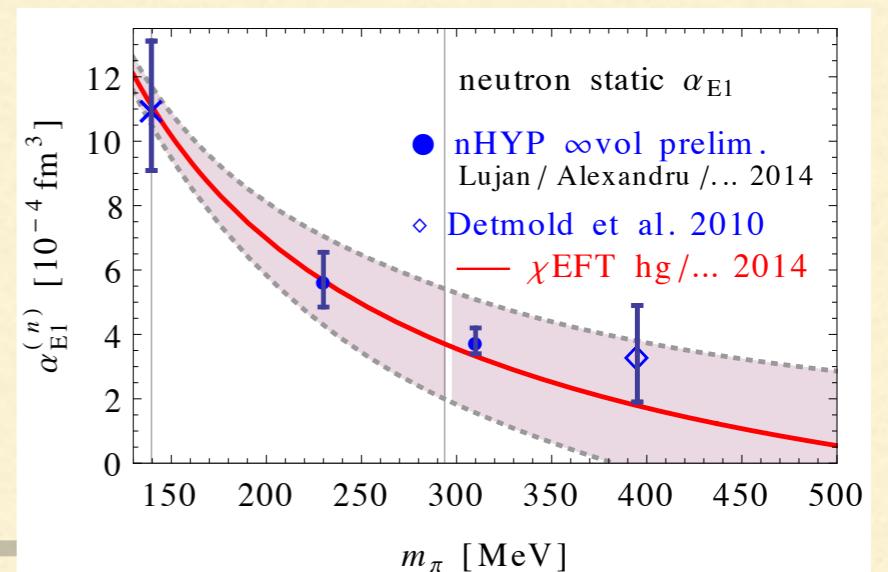
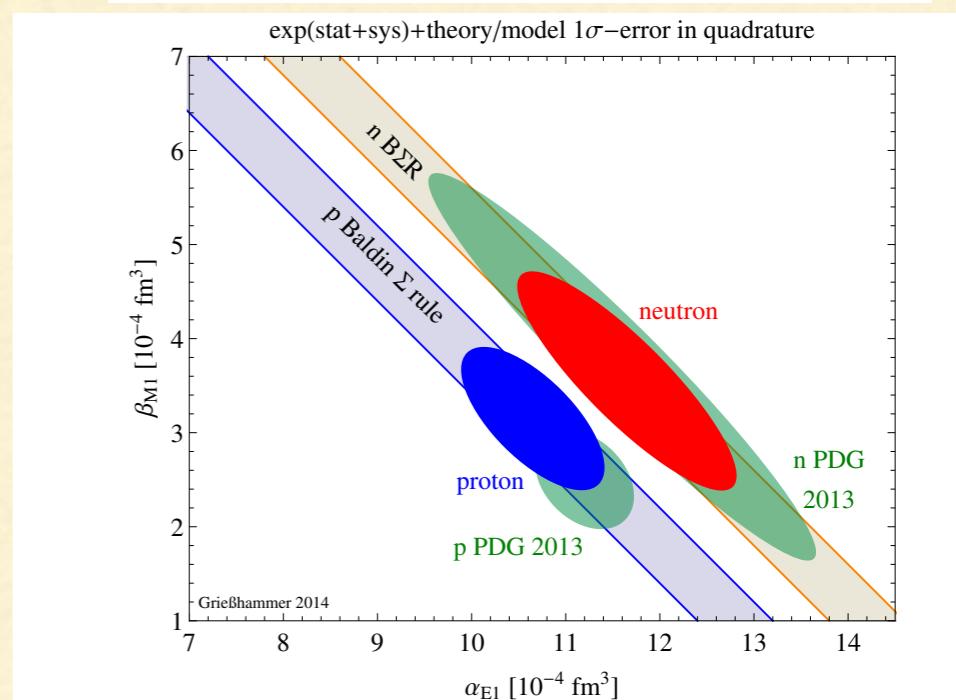
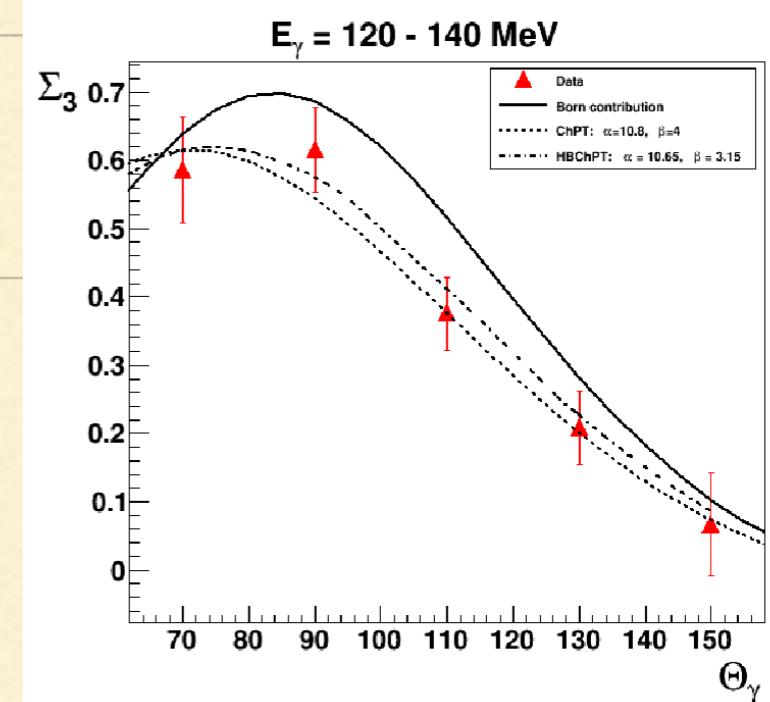
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LQCD
for
nuclei **EFT** Laboratory
measurements



BROAD THEMES

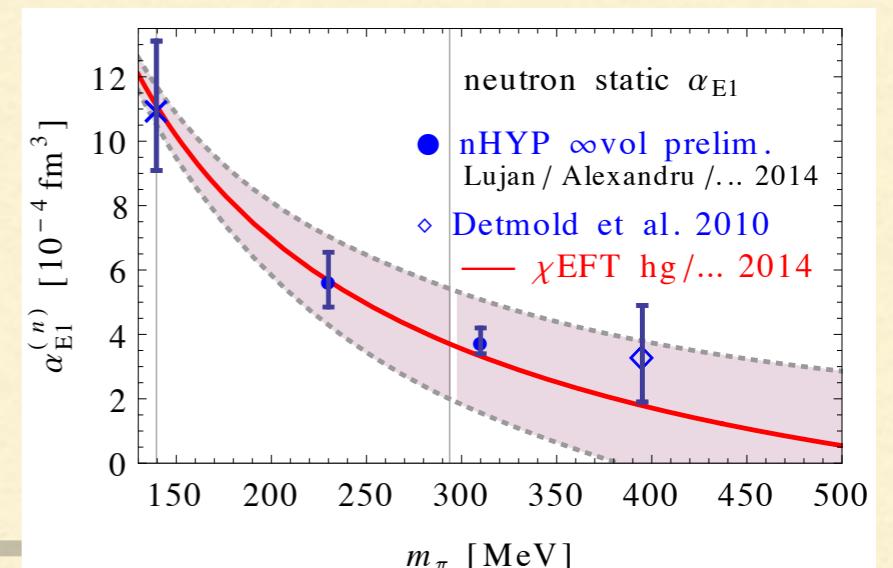
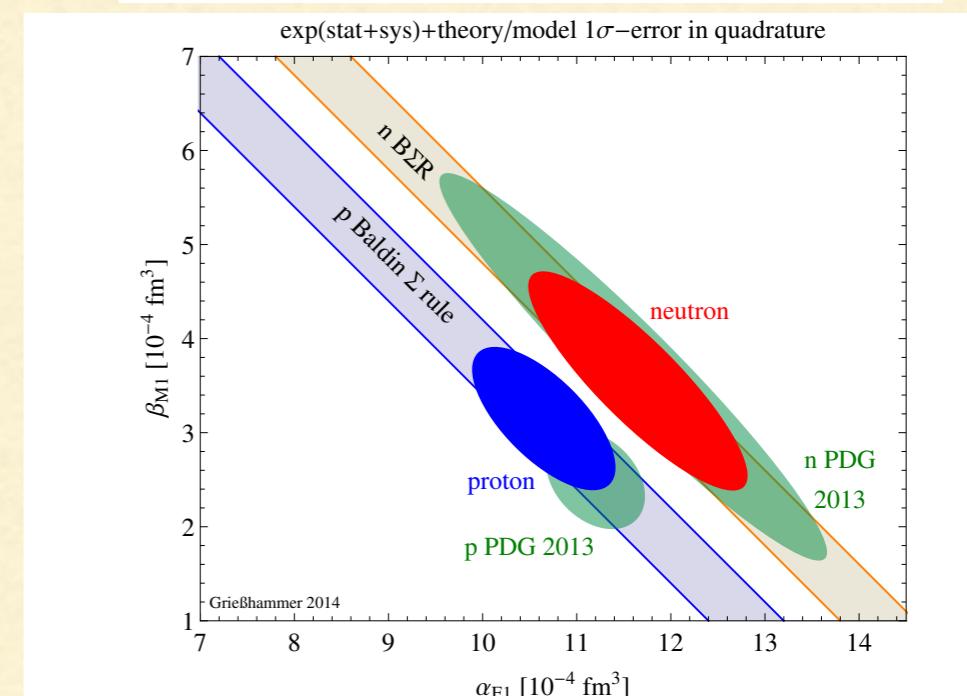
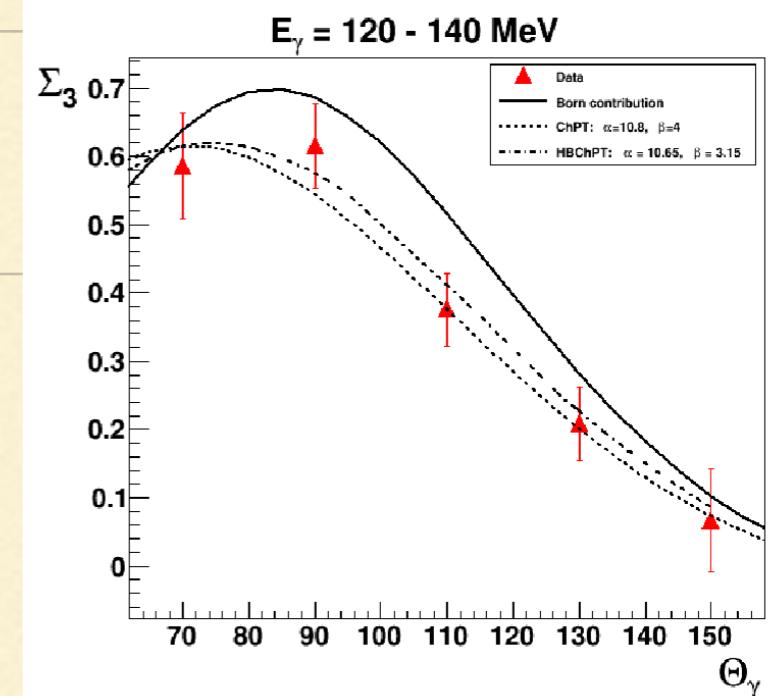
Picture credits: E. Downie, H. Grießhammer, A. Alexandru et al.



BROAD THEMES

Picture credits: E. Downie, H. Grießhammer, A. Alexandru et al.

- Theory: synergistic blend of LQCD, EFT, phenomenology
- Understand QCD interplay of π , Δ , heavier d.o.f.
- Strong emphasis on uncertainty quantification
- New experimental facilities and capabilities provide new opportunities to observe this interplay



BROAD THEMES

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- Theory: synergistic blend of LQCD, EFT, phenomenology
- Understand QCD interplay of π, Δ , heavier d.o.f.
- Strong emphasis on uncertainty quantification
- New experimental facilities and capabilities provide new opportunities to observe this interplay
- Theorists and experimentalists working together to identify best opportunities and interpret data
- Improvement in understanding of nuclear dynamics from χ EFT (and ultimately LQCD) allows nuclei to be used as neutron targets in novel ways.
- Chiral dynamics in $A=0, 1, 2, \dots$ now a precision tool: particularly important for BSM-physics searches

